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Forecasting VaR with Real-Time Realized EGARCH Model

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Abstract

In this paper, we propose a Real-Time Realized EGARCH model for forecasting the Value-at-Risk (VaR) in the Chinese stock market, utilizing a skewed t-distribution. This model introduces a novel approach by incorporateing high frequency intraday information and current return information into daily VaR forecast. We conduct an empirical analysis using two main indexes of Chinese stock market (Shanghai Stock Exchange Composite Index and Shenzhen Stock Exchange Component Index), and backtesting approach as well as the model robustness test prove that the VaR forecasts of Real-Time Realized EGARCH model generally gain an advantage over the GARCH model, the Real-Time GARCH model and the Realized EGARCH model. Our empirical findings highlight the value of incorporating both the high-frequency intraday information and current return information for forecasting the VaR of Chinese stock market.

Keywords

High-Frequency Intraday Information; Current Return Information; Real-Time Realized EGARCH Model; Var Forecast; Skew-T Distribution.

1. Introduction

In financial capital markets, since volatility is widely used as a proxy for the risk associated with financial assets and exhibits very rich and complex char-acteristics. Such as volatility clustering, time-varying and asymmetry. Reliable estimation and forecasting of volatility is therefore crucial for risk management. There are several main approaches to modeling the volatility and Value at Risk (VaR) of discrete financial time series. For example, generalized autoregressive conditional heteroskedasticity (GARCH) models.

The GARCH models assume that the volatility of asset returns is driven by historical information only, SV model assume that the volatility of asset returns is driven by the history of returns as well as by the history of unobserved random shocks. The difference in the incorporated information structure is also in their nature: While GARCH models incorporate only past internal determination of information, SV models consider volatility to be generated by an implied stochastic process that are independent from the shocks governing the returns process. As a result, the parameter-driven SV models are more flexible than the GARCH models in fitting financial time series data; however, this comes at the cost of higher complexity involved in their inference and estimation. In contrast, the observation-driven GARCH models have the advantages of a simple structure and easy to implement. As a consequence, the GARCH models have gained popularity in the literature, and forecasting the Chinese stock market volatility and market risk mainly relies on the GARCH models.

Nevertheless, the conventional GARCH and SV models only utilize daily return data to measure volatility and market risk. However, daily yield data only reflect historical characteristics of asset prices and neglect the high-frequency intraday information. When financial markets are in a period of high volatility, traditional GARCH models are unable to adjust quickly to market

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changes, and the extracted volatility information contains strong noise, resulting in market risk measurement errors.

In addition to capturing asymmetric volatility, it's critical to reasonably assume the distribution of returns to accurately predict VaR. Financial returns often exhibit skewness, leptokurtosis, and fat tails, characteristics that traditional distributions like the normal distribution do not adequately capture. In recent years, many researchers have used the Student-t distribution to model the thickness of the tails. However, this distribution does not effectively capture the skewness of returns.

Subsequent research has introduced skewed thick-tailed distributions, such as the skewed t-distribution (Hansen et al., 2011)[1]. Due to its flexibility and ability to model both skewness and fat tails, the skewed t-distribution is widely used and has demonstrated superior risk prediction performance(Trabelsi and Tiwari, 2023; Liu et al.,2024)[2-3].

The main contributions of this paper are delineated as follows. Firstly, we introduce a novel volatility model, specifically the Real-Time Realized EGARCH model, which adeptly incorporates high-frequency intraday information along with current return data to model and forecast VaR in the Chinese stock market.

Secondly, we take into account the characteristic features displayed by the distribution of asset returns, which typically exhibit skewness, leptokurtosis, and fat tails. To accurately capture the distribution of asset returns, we introduce the Skewed Student's t-distribution. By utilizing the Skewed Student's t-distribution, we can better model the tail risk and asymmetric behavior inherent in asset returns.

Thirdly, our proposed Real-Time Realized EGARCH model is rigorously applied to the Shanghai Stock Exchange Composite Index (SSEC) and the Shenzhen Stock Exchange Component Index (SZSEC) in China. Furthermore, posterior analysis methods are utilized to statistically evaluate the accuracy of VaR estimations. Empirical evidence indicates that the Real-Time Realized EGARCH model delivers more precise out-of-sample VaR predictions compared to conventional GARCH, Real-Time GARCH, and Realized EGARCH models.

Finally, we consider the out-of-sample VaR forecasting performance under different sample intervals. Different sample intervals affect the model's parameter estimation and the out-of-sample VaR forecasting performance. Therefore, we consider including both out-of-sample periods of extreme volatility (the 2015 'crash period') and periods of market stabilisation to enhance the consistency and robustness of the empirical results.

The remainder of the paper is organized as follows. In Section 2, we introduce the Real-Time Realized EGARCH model. It gives the outlines of our proposed model and the overview of Skew-t distribution, describes the mechanism of Real-Time Realized EGARCH model based on skew-t distribution and its maximum-likelihood (ML) estimator. Section 3 outlines VaR forecasting and backtesting.We have updated the formula for calculating VaR and introduced a test to evaluate the predictive accuracy of our model. Section 4 presents our data set, competing models and estimation results, examines the out-of-sample forecast performance of our proposed model and robustness test. Finally, Section 5 concludes the paper.

2. Methodology

2.1. Real-Time Realized EGARCH model

The REGARCH model is an improved version of the RGARCH model proposed by Hansen et al. (2012), which introduces two leverage functions to capture the leverage effect more flexibly. Hansen and Huang (2016) found that the REGARCH model exhibits better data fitting performance compared to the RGARCH model. On this basis, we introduce Real-Time Realized EGARCH model. Compared to the REGARCH model, this model adds $\gamma \log\varepsilon^2{}_t$ to Eq. (2). Whereas

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ε^2_t is a standardized innovation term, and the parameter γ quantifies the effect of the current return information on volatility. Such a structure has two main advantages, firstly, it avoids the difficulty of solving the model due to the direct inclusion of the current information term, and secondly, it takes into account the shock to volatility due to current information through a simple model.

The Real-Time Realized EGARCH model proposed in this paper is given by

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \mu + \sigma_t\varepsilon_t, \quad \varepsilon_t\,|F_{t-1} \sim \text{i.i.d.}N\,(0,1) \tag{1}$$

$$\log\sigma_t^2 = \omega + \beta\log\sigma_{t-1}^2 + \gamma\log\varepsilon_t^2 + \delta\,(\varepsilon_{t-1}) + \alpha u_{t-1} \tag{2}$$

$$\log x_t = \xi + \phi\log\sigma_t^2 + \tau(\varepsilon_t) + u_t, \quad u_t|F_{t-1} \sim \text{i.i.d.}N\left(0,\sigma_u^2\right) \tag{3}$$

where r_t is the log-return on day t, P_t is the closing price on day t, μ is the conditional mean of the returns, σ_t is the conditional volatility of the returns. Let x_t denote the realized measure of volatility on day t, and F_{t-1} be the information set up to day $t-1$.The two shocks, ε_t and u_t, are assumed to be independent. $\delta(\varepsilon_t)$ and $\tau(\varepsilon_t)$ are the leverage function, capturing the asymmetry in volatility (the asymmetric reaction of volatility to bad news ($\varepsilon_t < 0$) versus good news ($\varepsilon_t > 0$), and they take a quadratic form:

$$\delta(\varepsilon_t) = \delta_1\varepsilon_t + \delta_2(\varepsilon^2_t - 1) \tag{4}$$

$$\tau(\varepsilon_t) = \tau_1\varepsilon_t + \tau_2(\varepsilon^2_t - 1) \tag{5}$$

They satisfy $E[\delta(\varepsilon_t)] = E[\tau(\varepsilon_t)] = 0$.The equations (1)-(3) are respectively denoted as the return equation, the volatility equation, and the measurement equation. It merits emphasis that the volatility equation (2) is analytically tractable and can be equivalently expressed as follows:

$$\log\sigma_t^2 = \frac{\omega + \beta\log\sigma_{t-1}^2 + \gamma\log\left(r_t-\mu\right)^2 + \delta(\varepsilon_{t-1}) + \alpha u_{t-1}}{1+\gamma} \tag{6}$$

From Eq. (6), it becomes apparent that $\log\sigma_t^2$ is influenced by the current return information r_t and can be updated contingent upon σ_{t-1}^2, r_t , ε_{t-1} and μ_{t-1}. The parameter α signifies the information of the realized measure of volatility concerning future volatility, whereas the parameter γ quantifies the effect of the current return information on volatility. To ensure the stationarity of the volatility process, we enforce the condition that $|\beta| < 1$. Consequently, we ascertain that $E\left[\log\sigma_t^2\right] = \frac{\omega+\gamma(\log(2)+\psi(1/2))}{1-\beta}$, where $\psi(\cdot)$ denotes the digamma function.

The measurement equation (3) establishes the relationship between the realized measure of volatility and the underlying true volatility. The bias-correction coefficients, ξ and ϕ , are devised to rectify the bias in the realized measure of volatility attributable to non-trading hours and market microstructure noise. In practice, $\phi = 1$ is often assumed to simplify the model structure and to improve the out-of-sample fit(see, e.g., Hansen and Huang, 2016; Wu et al., 2020).

In this paper, we opt for the straightforward realized volatility (RV) as the realized measure of volatility, denoted by x_t, which is computed as

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$$RV_t = \sum_{i=1}^{N} r_{i,t}^2 \tag{7}$$

Where $r_{i,t}$ is the i th intraday log-return on day t, and N is the number of intraday log-returns, which is dependent on the sampling frequency. In this case, we utilize 5-minute high-frequency intraday data to compute RV. Consequently,

we have $N = 48$.

Notably, our proposed Real-Time Realized EGARCH model enables the effective utilization of high-frequency intraday information alongside current return information, which provides an advantageous structure for modelling and forecasting stock market VaR.

2.2. Skew-t Distribution

In order to reasonably measure financial market risks, the distribution setting of asset returns is critical. The traditional Realized EGARCH model assumes that the innovation of asset return follows a standard normal distribution.

2.3. Maximum Likelihood Estimation

The parameters of the Real-Time Realized EGARCH model are estimated via the maximum likelihood method, utilizing data on stock returns and the realized measure. Precisely, the joint log-likelihood function of the model is formulated as

$$\ell(r,x;\Theta) = \underbrace{\sum_{t=1}^{T} \log f(r_t \mid F_{t-1})} + \underbrace{\sum_{t=1}^{T} \log f(x_t \mid r_t, F_{t-1})} \tag{8}$$

where $\Theta = (\mu,\omega,\alpha,\beta,\gamma,\xi,\phi,\delta_1,\delta_2,\tau_1,\tau_2,\sigma_u)$ is the vector of model parameters, $\ell(r;\Theta)$ and $\ell(x|r;\Theta)$ denote the partial log-likelihood functions of returns and realized measure, respectively. The partial log-likelihood function of returns, $\ell(r;\Theta)$, can be written as

$$\ell(r;\Theta) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log 2\pi + d_{1,t}^2 - \log d_{2,t}^2\right] \tag{9}$$

Where

$$d_{1,t} = sign\,(r_t-\mu)\, e^{\left(\dfrac{\log\,(r_t-\mu)^2 - b_{t-1}}{2+2\gamma}\right)} \tag{10}$$

$$d_{2,t} = \frac{1}{(1+\gamma)|r_t-\mu|} e^{\left(\dfrac{\log\,(r_t-\mu)^2 - b_{t-1}}{2+2\gamma}\right)} \tag{11}$$

Where $b_{t-1} = \omega + \beta \log \sigma_{t-1}^2 + \delta(\varepsilon_{t-1}) + \alpha u_{t-1}$.

The partial log-likelihood function of realized measure, $\ell(x|r;\Theta)$, can be written as

$$\ell(x|r;\Theta) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log(2\pi) + \log\sigma_u^2 + u_t^2/\sigma_u^2\right] \tag{12}$$

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where $u_t = \log x_t - (\xi + \phi \log \sigma_t^2 + \tau(\varepsilon_t))$.
Given the joint log-likelihood function, $\ell(r,x;\Theta)$, the maximum likelihood estimates of the model parameters can be obtained via

$$\Theta =b \quad \underset{\Theta}{\operatorname{argmax}} \ell(r,x;\Theta) \tag{13}$$

3. VaR Forecasting and Backtesting

3.1. VaR Forecasting

The accurate measurement and prediction of financial risk hold significant importance for financial institutions, as they help prevent potential financial hazards. VaR as a crucial metric for assessing financial risk, has been endorsed by the Basel Committee since the 1990s and is now widely utilized in the risk management practices of financial institutions because of its intuitive nature, simplicity. VaR is defined as the maximum loss that assets or their portfolios may suffer within a specific future period, under a given confidence level.According to the definition, the VaR at time $t + 1$ for a given confidence level (1-α) can be represented as:

$$P\,(r_{t+1} > \mathrm{VaR}_{t+1}(\alpha)|F_t) = 1 - \alpha \tag{14}$$

For a long position, the risk primarily arises from a decrease in the asset price. Consequently, the VaR defined in Eq. (17) represents a negative value, indicating the maximum potential loss at a certain confidence level α, which is usually a small probability. According to the definition of VaR in Eq. (17), the VaR under the GARCH and REGARCH models can be formulated as:

$$\mathrm{VaR}_{t+1} = \mu + \varepsilon_\alpha \sqrt{h_{t+1}} \tag{15}$$

Where ε_α denotes the α quantile for the Skew-t distribution. However, Eq. (18) is not valid for Real-Time GARCH and Real-Time Realized EGARCH models.
Inspired by Ding (2023),We recalculate the VaR using the Eq. (17).

$$\begin{aligned} P\,(r_{t+1} > \mathrm{VaR}_{t+1}\,|F_t) &= P\,(\mu + \varepsilon_t\sigma_t > \mathrm{VaR}_{t+1}\,|F_t) \\ &= p(\varepsilon_t > d_1\,(\mathrm{VaR}_{t+1})|F_t) = 1 - \alpha \end{aligned} \tag{16}$$

where $d_1(\cdot)$ is given in Eq. (13).Therefore, the VaR for Real-Time GARCH Can be obtained by inverting Eq. (19).

$$\begin{aligned} \mathrm{VaR}_{t+1} &= d_1^{-1}(F_\varepsilon^{-1}(1-\alpha)) \\ &= \mu - \sqrt{(\omega + \beta_t\sigma_t^2 + \alpha\sigma_t^2\varepsilon_t^2)(F_\varepsilon^{-1}(1-\alpha))^2 + \gamma(F_\varepsilon^{-1}(1-\alpha))^4} \end{aligned} \tag{17}$$

The VaR for Real-Time Realized EGARCH can be formulated as

$$\begin{aligned} \mathrm{VaR}_{t+1} &= d_1^{-1}(F_\varepsilon^{-1}(1-\alpha)) \\ &= \mu - \exp(\frac{1}{2}b_t + \frac{1}{2}(\gamma+1)\log(F_\varepsilon^{-1}(1-\alpha))^2) \end{aligned} \tag{18}$$

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where $b_t = \omega + \beta \log\sigma_t^2 + \delta(\varepsilon_t) + \alpha u_t$.

3.2. Backtesting

To assess the accuracy of VaR forecasts, we employ the backtesting methods that are widely used in academic literature, specifically focusing on two tests: the failure rate test and the likelihood ratio test for unconditional coverage, as introduced by Kupiec (1995). Specifically, the loss sequence is defined as:

$$LS_{t+1} = \begin{cases} 1, & if\ r_{t+1} \leq VaR_{t+1} \\ 0, & if\ r_{t+1} \geq VaR_{t+1} \end{cases} \tag{19}$$

The likelihood ratio statistic of the unconditional coverage test, which evaluates whether a model's predictions are consistent with the observed frequency of exceedances, is given by the following formula:

$$LR = -2\ln[(1-\alpha)^{T_0}\alpha^{T_1}(1-\pi)^{-T_0}\pi^{-T_1}] \sim \chi^2(1) \tag{20}$$

Where T_0 is the number of times $LS_{t+1} = 0$, T_1 is the number of times $LS_{t+1} = 1$, and $\pi = \frac{T_1}{T_0+T_1}$ is the failure rate.

4. Empirical Analysis

4.1. Data

For our empirical analysis, we use data on the 5-minute high-frequency intraday and daily log-returns of SSEC and SZSEC of China. The RV is calculated based on the 5-minute intraday log-returns by Eq. (7). The sample spans the period from January 4, 2005 to December 29, 2023, resulting in a total of 4615 daily observations for the daily log-returns and RV series. The data are obtained from the Wind Database of China.

Table 1. Descriptive statistics of daily returns, RV and log-RV for SSEC and SZSEC.

	Mean	Min.	Max.	Std.	Skewness	Kurtosis	J-B
				SSEC			
rt	0.0002	-0.0926	0.0903	0.0152	-0.5837	8.1702	5402.1443
RV	0.0002	0.0000	0.0039	0.0003	6.1308	61.0712	67369.6508
log-RV	-9.4284	-12.0260	-5.5450	1.0322	0.5838	3.0840	263.4663
				SZSEC			
rt	0.0002	-0.0975	0.0916	0.0176	-0.4991	6.3078	2295.0678
RV	0.0002	0.0000	0.0053	0.0003	5.8546	58.7859	624652.9947
log-RV	-9.0327	-11.8412	-5.2414	0.9614	0.4017	3.1097	126.4255

Note: This table reports the Descriptive statistics of daily returns, RV and log-RV for SSEC and SZSEC for the sample period from January 4, 2005 to December 29, 2023. J-B denotes the Jarque-Bera statistic.

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Table 1 reports the descriptive statistics of the daily returns, RV and logRV for the SSEC and SZSEC. As shown in Table 1, the return distributions for the SSEC and SZSEC exhibit negative skewness and excess kurtosis. The Jarque-Bera statistics indicate that the return distributions deviate from the Gaussian distribution. The RV distributions for the SSEC and SZSEC are also non-Gaussian with positive skewness and excess kurtosis. It is important to note, however, that the distributions of log-RV series are close to Gaussian distribution, which is consistent with the finding in Figure 3.

4.2. Out-of-sample VaR Forecasts

As good in-sample performance does not guarantee good out-of-sample forecasting, we focus on assessing the practical predictive capabilities of our models. To do so, we divide the dataset into two parts: an in-sample estimation period from January 4, 2005, to April 24, 2015 (2500 observations within the sample), and an out-of-sample forecast evaluation period spanning from April 27, 2015, to December 29, 2023 (2115 observations out of sample). The latter period was selected because it corresponds to a time of significant volatility in Chinese stock markets, resulting in substantial losses for many investors. This makes it a critical period for testing the reliability and accuracy of risk models in realworld conditions. For out-of-sample forecasting, we implement a rolling window procedure.

we present the results of backtesting, namely the failure rate test and the likelihood ratio tests of unconditional coverage (LR) of VaR forecasts with the probabilities $1-\alpha = 0.99, 0.975, 0.95, 0.9$ for the SSEC and SZSEC, using the GARCH, Real-Time GARCH, Realized EGARCH and Real-Time Realized EGARCH. We observe from the table that Real-Time Realized EGARCH has best forecasting performance for confidence levels of 99% , 97.5% and 90%, and according to the LR tests, Real-Time Realized EGARCH passes the LR test in all cases. However, the Real-Time GARCH and Realized EGARCH models are generally rejected at the 10% significance level for the case of 5% VaR.

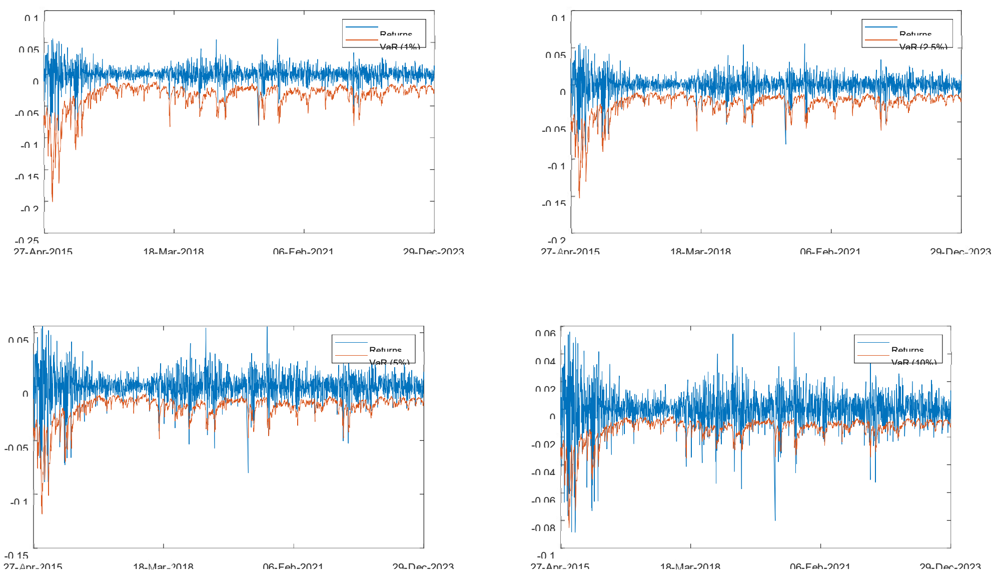

Figure 1. VaR Forecast Results for the SSEC.

Overall, the GARCH, Real-Time GARCH, and Realized EGARCH models tend to produce more conservative VaR estimates than the Real-Time Realized EGARCH model, which might underestimate risks. In summary, the RealTime Realized EGARCH model outperforms others in terms of VaR estimation across all confidence levels, as evidenced by passing the likelihood ratio tests. Its superiority is particularly pronounced at extreme confidence levels (99% confidence level), where it notably improves VaR estimation accuracy. This underscores the

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critical role of high-frequency and current return information into the Real-Time Realized EGARCH model for enhancing VaR estimation accuracy, especially in critical risk situations.

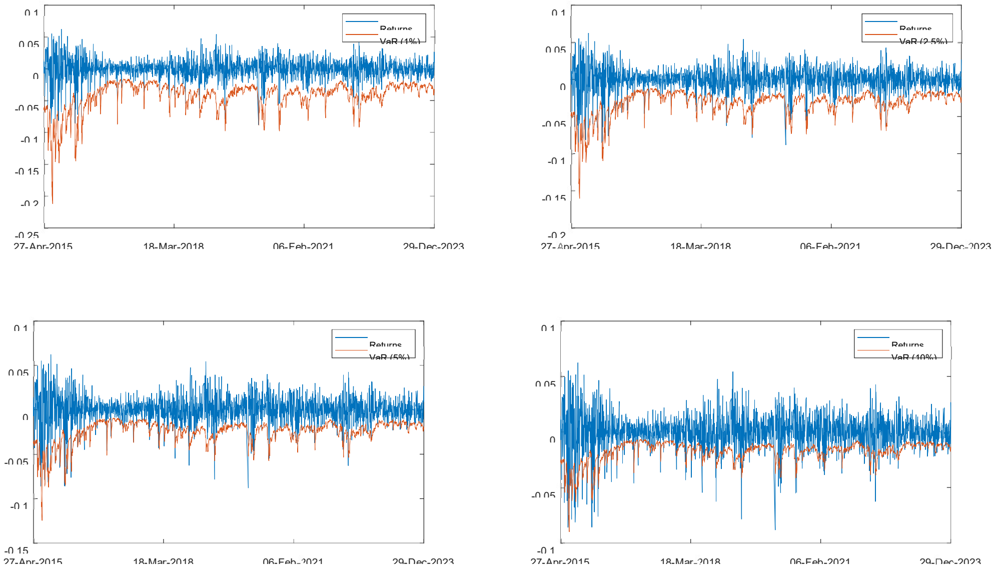

Figure 2. VaR Forecast Results for the SZSEC.

5. Conclusion

Effective measurement of price risk in the stock trading market is of practical importance for market participants. In this paper, we propose the Real-Time Realized EGARCH model to model and forecast the Chinese stock market VaR. The proposed model can be viewed as a model combination of the Real-Time GARCH and Realized EGARCH models. As a consequence, the Real-Time Realized EGARCH model is able to accounting for the high-frequency intraday information and current return information simultaneously.

We apply the Real-Time Realized EGARCH model to the SSEC and SZSEC data, and considering the characteristic features displayed by the distribution of asset returns. We also test the robustness of the model by varying the sample interval. Our empirical results show that the Real-Time Realized EGARCH model outperforms the GARCH model, the Real-Time EGARCH model and the Realized EGARCH model in out-of-sample VaR forecast.

Overall, our empirical findings provide strong support for incorporating highfrequency intraday information and the current return information to improve VaR forecasts. A natural line of future research would be to further extend our model by incorporating option-implied information. Additionally, it would be meaningful to apply our model to different financial applications, such as option pricing. we leave this in our future work.

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