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Portfolio Dynamic Optimisation based on GARCH Volatility Forecasting

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Abstract

This study focuses on the portfolio optimisation problem in the financial market, aiming to construct a portfolio model that integrates GARCH volatility prediction and dynamic optimisation. By collecting daily frequency data of five weighted stocks in the S&P 500 constituents, this article use GARCH (1,1) and EGARCH models to predict asset volatility and design a rebalancing strategy by combining dynamic programming and quadratic programming. The study addresses the key issues of coupling the GARCH prediction results with the dynamic optimisation model, optimising the efficiency of quadratic programming under high-dimensional data, and verifying the stability of the dynamic strategy. The results show that the constructed model outperforms the static strategy in terms of return-risk performance, provides investors with more effective risk management tools, and enriches the theory and practice of financial risk management and asset allocation.

Keywords

Garch Model; Dynamic Optimisation; Portfolio; Volatility Forecasting.

1. Introduction

In today's financial world, volatility forecasting, as a key tool for financial risk management and asset allocation, can directly affect the balance of return and risk in asset allocation, and plays a pivotal role in investor decision-making. Accurate volatility forecasting can help investors dynamically adjust their investment portfolios to achieve a fine balance between return and risk. For example, in the case of diversified assets such as the S&P 500 constituents, the market environment is changing rapidly and it is often difficult for a static investment strategy to keep up with the pace of the market, whereas a dynamic rebalancing strategy can respond to market fluctuations in a more flexible manner. Dynamic rebalancing strategies can adjust asset weights in real time according to market fluctuations, avoiding the lag of static strategies and thus improving the return-to-risk ratio of the investment portfolio. However, most of the current studies focus on static optimisation or only use a single volatility model, and the practical application of dynamic optimisation combined with GARCH forecasting is still under-explored. This study aims to fill this gap by exploring the combination of dynamic optimisation and GARCH forecasting to create a more efficient risk management tool for institutional investors and quantitative traders. This research is expected to significantly improve investors' ability to cope with the complex and volatile financial markets and enhance the scientific and accurate investment decisions, thus effectively reducing investment risks and increasing investment returns.

This research plan adopts the GARCH model to predict volatility and combines dynamic programming and stochastic optimisation methods to construct a dynamic portfolio optimisation model.The GARCH model can effectively capture the volatility clustering of financial time series, while the dynamic optimisation method can adjust the asset allocation in real time according to the market changes to enhance the adaptability of the model. In the field of financial research, the use of this method is crucial, which can break through the constraints of the traditional linear assumptions and better adapt to the complex, variable and non-linear market environment. However, the current research in this area is still weak, mostly focusing on the prediction or static optimisation of a single model, and lacks comprehensive research combining GARCH prediction and dynamic optimisation. The main reason for this is that it is difficult to coordinate the suitability of different optimisation methods and there is a lack of effective means to dynamically adjust the model parameters. Existing studies mostly focus on the application of a single method, or the simple splicing of different methods, failing to give full play to the advantages of the synergy of multiple methods. Through this study, it is expected to further explore the potential of these methods, promote the innovation and development of financial risk management methods, and provide more solid theoretical support for the stable operation of the financial market.

From the academic level, this study will enrich the theoretical system in the field of financial risk management and asset allocation, fill the research gaps in the application of dynamic optimisation and GARCH prediction, and provide new ideas and methods for the subsequent related research. From the perspective of practical impact, the research results can provide a strong basis for financial institutions to formulate more scientific and reasonable risk management strategies and promote the sound development of the financial industry, and at the same time, it can also help investors to improve the level of investment decision-making, optimise their personal investment behaviours, and achieve a better balance between returns and risks in the complex financial market.

2. Current Status of Research at Home and Abroad

2.1. Current Status of Domestic Research

Domestic empirical studies on GARCH models started in the stock market. Hainan Huang and Zhong Wei first applied the GARCH family model to the return analysis of the SSE index, and through the realised volatility assessment, they found that the GJR (1,1) model based on the skewed t-distribution had the best prediction effect, which laid the foundation for the establishment of the subsequent prediction model assessment system [1]. Yan Dingqi and Li Yufeng extended the research object to CSI 300 index and analysed it by using GARCH and other models with different distributions, and came to the conclusion that the GARCH(1,1) model based on Student's t-distribution predicts the best volatility performance of the index in the next two days, which further enriches the results of the application of the model in different indices [2].

In recent years, with the diversified development of financial markets, the application areas of related research have been expanding. Xiangyu Cui and Lanzhi Yang extended their research perspective to the commodity futures market and found that the performance of different volatility prediction models in dynamic investment strategies varies significantly, among which the precision matrix model outperforms the traditional sample covariance method in the Sharpe ratio index, which opens up a new dimension for the application of GARCH model in portfolio optimization [3]. Liu Qi constructed a hybrid model of GARCH model and BiLSTM network to predict stock prices for small and medium-sized enterprise (SME) stock data characteristics and used Bayes method to make decisions, and the empirical results showed that the method can improve the prediction accuracy and achieve the return target, which expands the boundaries of the application of GARCH model [4].

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2.2. Current Status of Foreign Research

In foreign research, since Bollerslev proposed the GARCH model, its theoretical framework and application boundaries continue to expand. Early research mainly focuses on the expansion of model parameters and optimisation of distributional assumptions, such as Palm, who systematically sorted out the improvement path of the GARCH model from the normal distribution to the Student's t-distribution with jump components and pointed out the unique advantages of the factor GARCH model in terms of theoretical interpretations and downscaling predictions [5].

As the study of financial market anomalies deepens, Fernández and Castano, using the Colombian stock market as an empirical subject, confirm the superiority of the asymmetric EGARCH model in capturing the "thick tail" characteristics of the market and avoiding symmetry bias, which promotes a paradigm shift in volatility modelling from the symmetric framework to the asymmetric mechanism [6]. The paradigm shift from symmetric framework to asymmetric mechanism in volatility modelling has been promoted. Since then, scholars have begun to pay attention to the time-varying characteristics of different forecasting models. Based on the comparison of China's stock index futures data, Lin finds that the GARCH model is outstanding in short-term volatility forecasting, while implied volatility is more reasonable in long-term forecasting by virtue of the advantage of holographic information of the option price, which provides an important basis for choosing time-varying methodology for volatility forecasting [7].

In recent years, the rise of machine learning technology, Mishra et al. return to traditional econometrics in the study of India's ESG index and verify the sustained explanatory power of the traditional GARCH model on the volatility characteristics of the emerging ESG market through comparative experiments between the GARCH and LSTM models, revealing the irreplaceability of financial econometrics models in specific scenarios [8].

2.3. Summary of the Current Status of Research

It seems that there are a lot of research results on GARCH models in financial markets both at home and abroad. Existing research covers all areas of financial markets, from stock indices to commodity futures, and from developed markets to emerging ESG areas; the research content includes the evaluation of model forecasting effect, application in different markets and combination with other models; and the research methodology involves the use of various types of GARCH models in combination with actual data for analyses and comparisons. However, there are some gaps in these studies, as there are insufficient studies on the combination of GARCH models with dynamic portfolio optimisation in dynamic market environments, and there are fewer studies on the real-time adjustment mechanism of the models under complex and volatile market conditions.

3. Research Methodology

3.1. GARCH Modelling

In this study, the volatility model is fitted using Python's arch library. arch library provides a rich implementation of the GARCH family of models, which can facilitate model parameter estimation and model selection. In the model selection process, the optimal parameters are chosen by the AIC criterion, which takes into account the model's goodness-of-fit and complexity, and is able to select the optimal model among many models, avoid overfitting phenomenon, and ensure that the GARCH model can accurately capture the volatility characteristics of the asset return series.

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3.2. Dynamic Optimisation

A mean - variance objective function is constructed to determine the optimal asset allocation by weighing the expected return and risk (variance) of the portfolio. In this study, the rolling rebalancing logic is embedded to achieve dynamic adjustment of portfolio weights. The rolling rebalancing strategy can periodically adjust the portfolio according to market changes, so that the portfolio always maintains the optimal allocation, and improves the portfolio's adaptability and return-risk performance in a dynamic market environment.

3.3. Robust Optimisation

In order to hedge the impact of parameter estimation errors on the portfolio optimisation results, a random perturbation term is introduced and a robust optimisation approach is used. Robust optimisation takes into account the uncertainty of the parameters, and by constructing a robust optimisation model, the portfolio can maintain a better performance in spite of parameter fluctuations. In the actual financial market, there are often errors in parameter estimation, and the robust optimisation method can effectively reduce the negative impact of these errors on investment decisions and improve the stability and reliability of the portfolio.

4. Research Content and Technical Route

4.1. Content of the Study

4.1.1. Data Collection and Processing

Collecting daily frequency return data of S&P 500 constituent stocks. The S&P 500 index covers 500 large listed companies in the United States, which is widely representative of the market, and its constituent stock data can reflect the overall operation of the U.S. stock market. The collected data are subjected to smoothness test and outlier treatment. Smoothness is an important prerequisite for time series analysis, and non-smooth data may lead to problems such as pseudo-regression, which affects the accuracy of the model; outliers will cause greater interference in the model estimation results, and identifying and dealing with outliers through reasonable methods can improve the quality of the data and ensure the reliability of the subsequent analysis.

4.1.2. Volatility Forecasting

GARCH(1,1) and EGARCH models are used to forecast asset volatility; GARCH(1,1) model is one of the most commonly used GARCH models, which is able to better capture the volatility clustering of the financial time series; the EGARCH model takes into account the leverage effect on the basis of the GARCH model, that is, the fall of asset prices is often accompanied by greater volatility, which is more in line with the actual situation of the financial market. The optimal volatility forecasting model is selected by comparing the forecasting results of different models and evaluating the forecasting accuracy.

4.1.3. Dynamic Optimization Modelling

Combining dynamic planning and quadratic programming to design rebalancing strategies and optimize weight allocation. Dynamic programming is an effective method for solving multi-stage decision-making problems, and in portfolio optimisation, it can make optimal decisions according to the market situation in different periods; quadratic programming is an optimisation method that solves the optimal solution of the quadratic objective function under certain constraints. By combining the two, the dynamic optimisation of portfolio weights can be achieved and the return-risk ratio of the portfolio can be improved.

4.1.4. Robust Optimization Modelling

Introducing stochastic perturbation terms and using robust optimization to hedge against parameter estimation errors. Robust optimisation constructs a more robust optimisation

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model by considering the uncertainty of the parameters. In this study, it is assumed that there exists a certain range of fluctuation in the model parameters, and the optimal solution is found within this range, so that the portfolio can still maintain a better performance when the parameters fluctuate.

4.1.5. Empirical Analysis

Comparing the Sharpe ratio and maximum retracement of static and dynamic strategies. The Sharpe ratio measures how much excess return the portfolio will generate for each unit of total risk, which is an important indicator for evaluating the performance of the portfolio; the maximum retracement reflects the maximum loss that the portfolio may face in a certain period of time, which can visually demonstrate the risk control ability of the portfolio. By comparing these indicators, the validity of the model is verified to provide investors with a more scientific basis for investment decisions.

4.2. Technical Routes

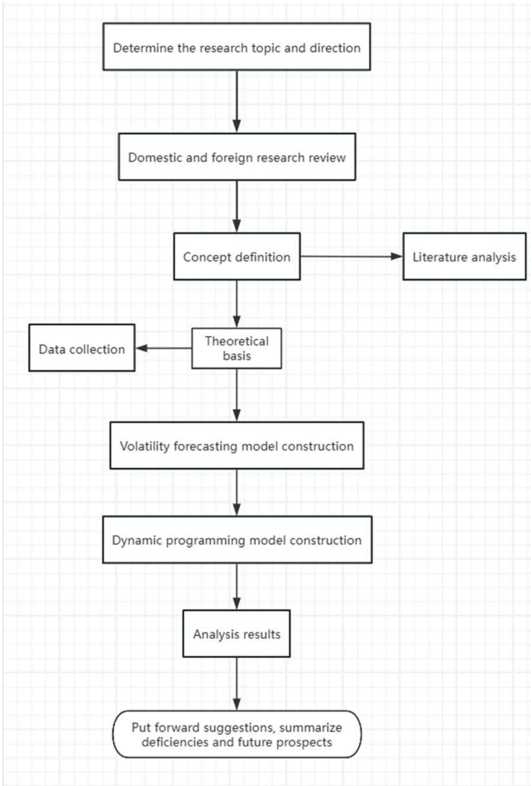

Fig. 1 research roadmap

Fig.1 commences with pinpointing the research topic and direction, followed by a comprehensive review of domestic and international research. Concept definition, informed by literature analysis, paves the way for establishing the theoretical basis, with data collection feeding into this stage. Subsequently, the construction of a volatility forecasting model and a dynamic programming model takes center - stage. After analyzing the results generated by these models, the process culminates in proposing suggestions, summarizing research shortcomings, and envisioning future prospects.

5. Modelling

5.1. GARCH Modelling

The general form of the GARCH (p, q) model is:

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$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (1)$$

whereσ_t^2 is the conditional variance, i.e., the square of volatility, at moment t;ω is the constant term;α_i andβ_j are the model parameters; andϵ_t is the residuals at moment t, obeying a normal distribution with mean 0 and varianceσ_t^2 . In this study, the focus is on using the GARCH(1,1) model, i.e., the case where p = 1 and q = 1, which takes the form:

$$\sigma_t^2 = \omega + a\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (2)$$

The GARCH(1,1) model succinctly captures the volatility clustering feature of financial time series, whereby past volatility has an impact on future volatility.

The EGARCH model, on the other hand, takes into account the asymmetry of the fluctuations and is expressed as:

$$\ln(\sigma_t^2) = \omega + \alpha \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta \ln(\sigma_{t-1}^2) \quad (3)$$

whereγ is the coefficient of the asymmetric term, which, when$\gamma \neq 0$, reflects the different effects of positive and negative shocks on volatility

5.2. Dynamic Planning Models

Use Python to construct a mean-variance objective function.

The objective function expression is:

$$maxE(R_p) - \frac{1}{2}\gamma Var(R_p) \quad (4)$$

where$E(R_p)$ is the expected return of the portfolio$Var(R_p)$ is the variance of the portfolio, which represents the risk, andγ is the risk aversion coefficient, which reflects the investor's preference for risk.

Embedded in the rolling rebalancing logic, the optimal weights of the portfolio are recalculated periodically based on new market data to achieve dynamic adjustment. At each rebalancing cycle, the portfolio's risk parameters are updated based on the volatility predicted by the GARCH model, which in turn adjusts the asset weights.

5.3. Robust Optimisation Model

Define the robust optimisation objective function as follows:

$$RobustObjective(w, r, \Sigma, \epsilon) = -\sum_{i=1}^{n} r_i w_i + \gamma \cdot w^T \Sigma w + \epsilon \cdot ||w|| \quad (5)$$

where w is a vector of asset weights, r is a vector of expected asset returns,Σ is the covariance matrix of asset returns,γ is the risk aversion coefficient, which reflects the degree of risk appetite of investors,ϵ is a robustness parameter, which is used to control the degree of influence of the robustness term, and n is the number of assets.

In the actual calculations, rolling windows are used to obtain asset return and volatility data for each period. For each rolling window:

(1). Calculate the covariance matrix from the return data within the windowΣ , the

The formula is

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$$\Sigma = diag(\sigma) \cdot Corr(R) \cdot diag(\sigma) \tag{6}$$

where $diag(\sigma)$ is a diagonal matrix consisting of the standard deviation of each asset's volatility,$Corr(R)$ is a matrix of correlation coefficients of asset returns, and R is a matrix of asset returns within the window.

(2). The robust optimisation problem is solved using an optimisation algorithm with the constraints that the sum of the asset weights is 1 and the weights are non-negative, i.e:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0, \quad i = 1, \dots, n \tag{7}$$

6. Empirical Analysis

6.1. Data Sources and Processing

The daily frequency return data of S&P 500 constituents from 2024.5.1 to 2025.5.1 are selected as the sample for this study. The data is obtained from Yahoo Finance API, which provides rich financial market data with high data quality. In the data processing stage, the collected yield data are firstly tested for smoothness, and ADF test and other methods are used to determine whether the data are smooth or not. If the data are not smooth, processing such as differencing is carried out to make them smooth. At the same time, outliers are identified and dealt with through statistical methods, such as setting reasonable thresholds, and data exceeding the thresholds are regarded as outliers to be corrected or deleted.

6.2. Volatility Forecast Results

The GARCH(1,1) sum model is applied to forecast asset volatility. The prediction accuracy of the model is assessed by prediction error metrics. Commonly used indicators include root mean square error (RMSE), mean absolute error (MAE).RMSE can reflect the average degree of error between the forecast value and the real value, and its calculation formula is:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \tag{8}$$

wherey_i is the actual value,$\hat{y}_i$ is the predicted value, and n is the sample size.

The MAE is calculated by the formula:

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i| \tag{9}$$

The smaller the RMSE and MAE values, the higher the predictive accuracy of the model.

By comparing the RMSE and MAE values of different models, the model with higher prediction accuracy was selected for subsequent dynamic optimisation modelling.

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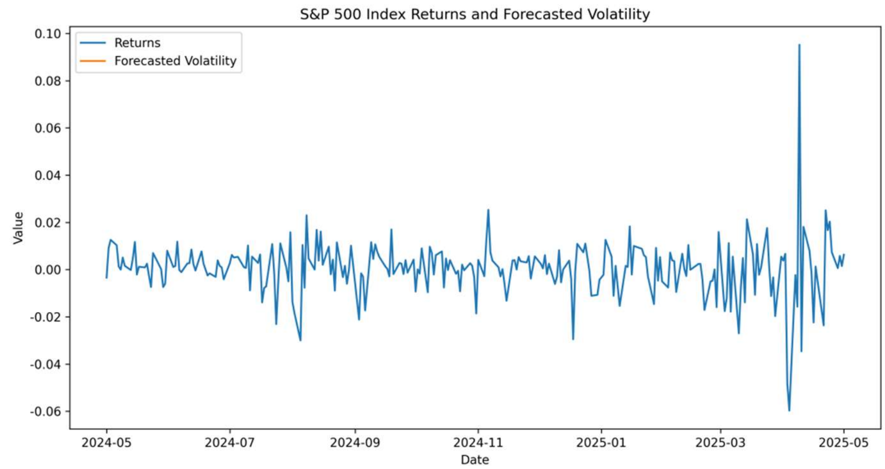

Fig. 2 2024.5.1-2025.5.1 S&P 500 Index Volatility by GARCH (1,1)

RMSE: 0.0030011300789703 09
MAE: 0.0030011300789703 09

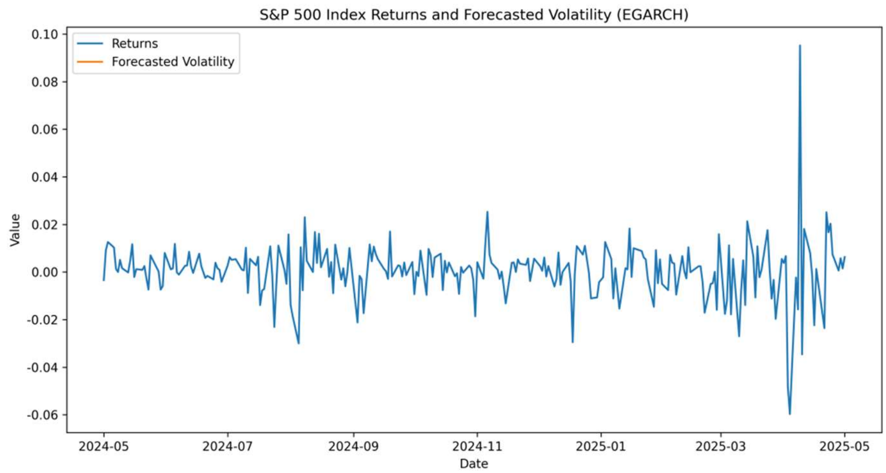

Fig. 3 2024.5.1-2025.5.1 S&P 500 Index Volatility by EGARCH

RMSE: 0.002982427815728599
MAE: 0.002982427815728599

The results of Fig.2 and Fig.3 show that in the in-sample prediction, the EGARCH model performs better in capturing the market leverage effect, and its predicted volatility fits the actual volatility better, with a relatively small RMSE value; while the GARCH(1,1) model has advantages in computational efficiency and simplicity. In the out-of-sample prediction, there are some differences in the prediction effect of the two models, and the EGARCH model predicts future volatility closer to the actual value when the market fluctuates more violently, which can better provide investors with risk warning.

The results of the five stocks selected (TSLA, NVDA, MSTF, GOOG, AAPL) are shown below:

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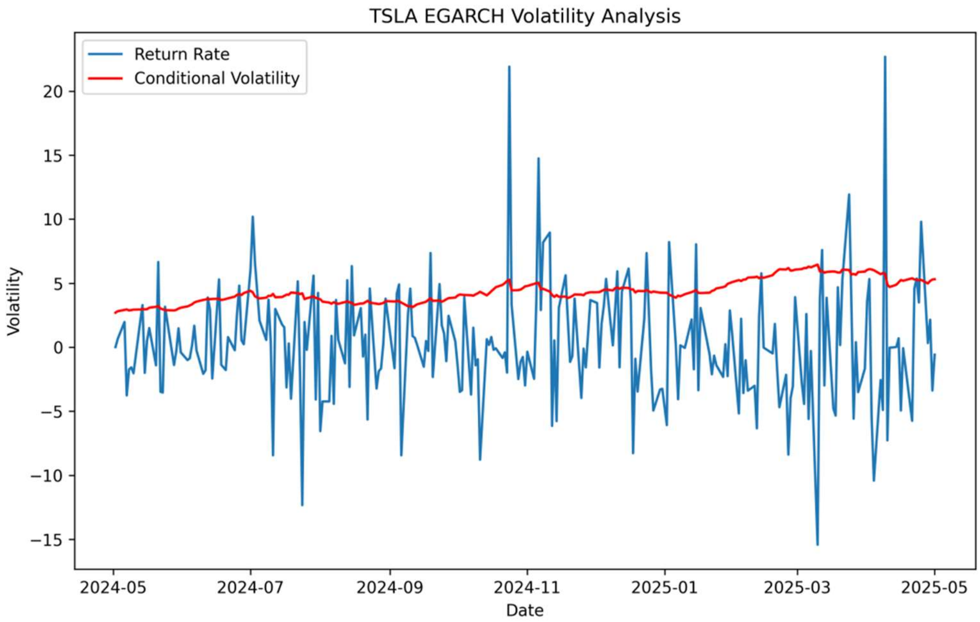

Fig. 4 2024.5.1-2025.5.1 TSLA stock price volatility by EGARCH

TSLA RMSE: 6.1907341352711835
tsla mae: 4.990626080331105

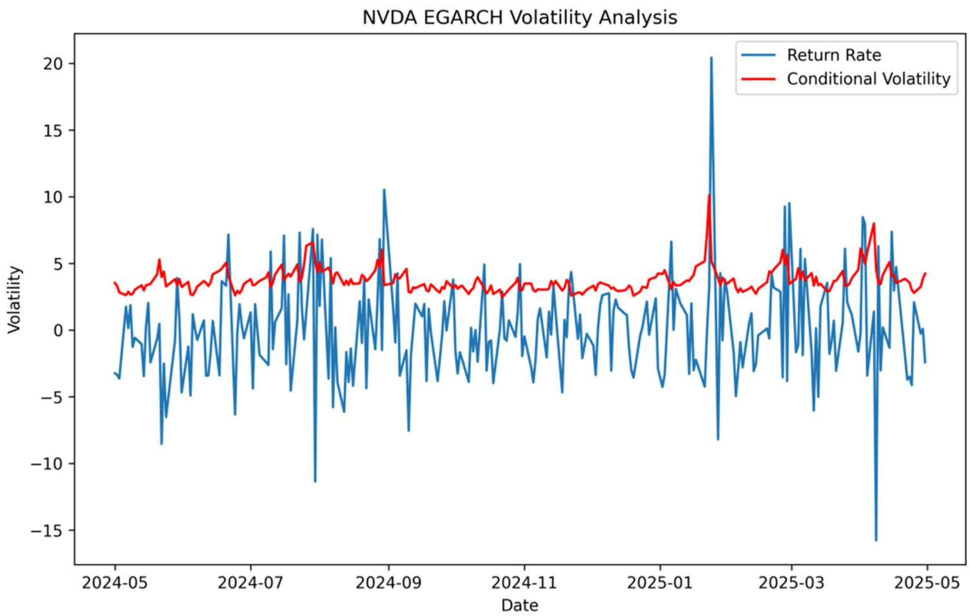

Fig. 5 2024.5.1-2025.5.1 NVDA stock price volatility by EGARCH

NVDA RMSE: 5.3329068515707805
nvda mae: 4.466402679033881

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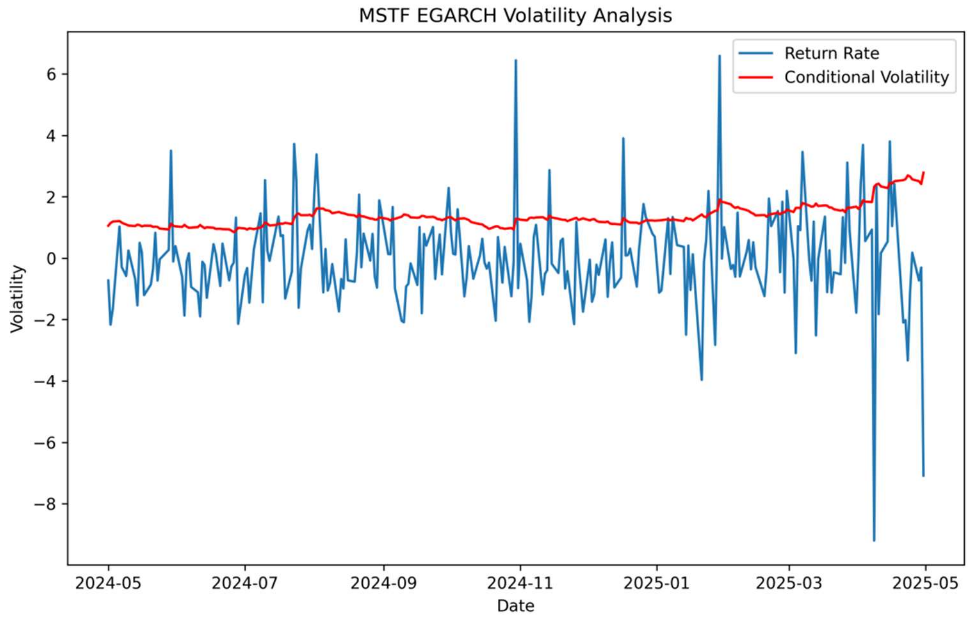

Fig. 6 2024.5.1-2025.5.1 MSTF stock price volatility by EGARCH

MSTF RMSE: 2.1636596628902263
MSTF MAE: 1.6869084352367485

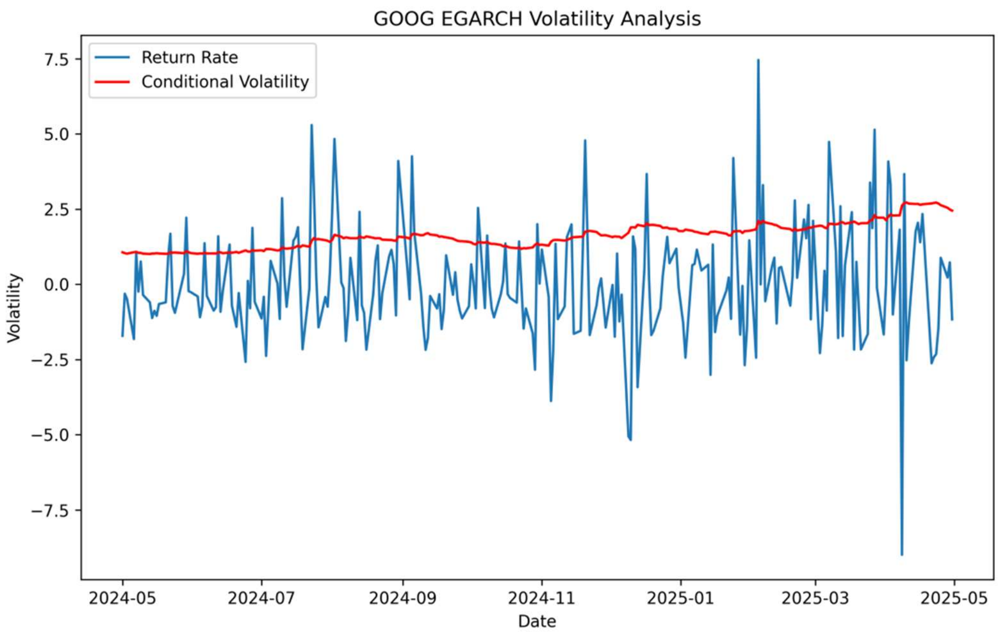

Fig. 7 2024.5.1-2025.5.1 GOOG stock price volatility by EGARCH

goog rmse: 2.47088666052412
goog mae: 2.0062596831013346

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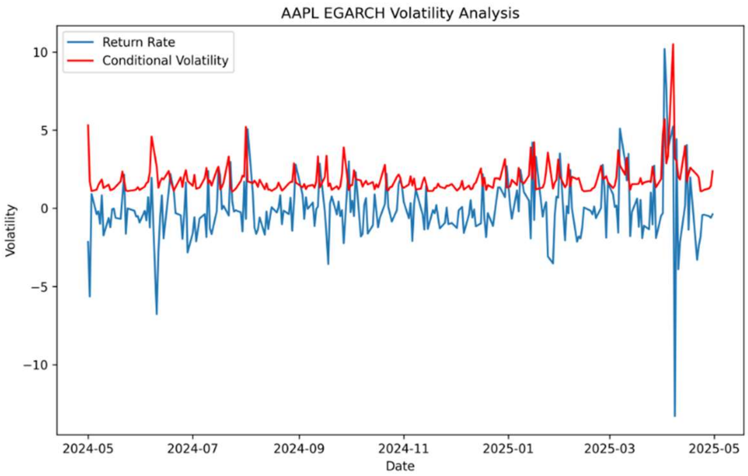

Fig. 8 2024.5.1-2025.5.1 AAPL stock price volatility by EGARCH

aapl rmse: 2.8285027173905477

aapl mae: 2.2634718439250383

6.3. Portfolio Optimisation Results

The dynamic portfolio optimisation model is constructed by combining dynamic planning and quadratic planning, and compared with the static portfolio strategy. Backtest analysis is conducted to calculate and compare the Sharpe ratio and maximum retracement index of the two strategies to verify the effectiveness of the model.

The Sharpe Ratio is a measure of how much additional return over the risk-free return a portfolio will generate for each unit of total risk it is exposed to, and is calculated as:

$$Sharpratio = \frac{E(R_p)-R_f}{\sigma_p} \tag{10}$$

Where$E(R_p)$ is the expected return of the portfolio,R_f is the risk-free rate of return andσ_p is the standard deviation of the portfolio, which represents the risk. The higher the Sharpe ratio, the higher the return the portfolio earns per unit of risk taken.

Maximum retracement is defined as the maximum value of the magnitude of the retracement of the rate of return when the NAV of the product goes to its lowest point at any historical point in a selected period, and is used as a measure of the extent of the portfolio's loss in the most unfavourable scenario.

Based on the portfolio NAV data$P_1, P_2, \dots, P_n$, for each point in time $i(1 \leq i \leq n)$, find the minimum value of the NAV from that point in time backwards$P_{\min(i)}$.

The retracement margins D_i at the point in time$i$ can be calculated using the formula$D_i = \frac{P_i - P_{\min(i)}}{P_i}$ The maximum retracement over the entire period MDD is the maximum of these retracement margins,

$$MDD = max(D_i) \tag{11}$$

By comparing the performance of static and dynamic strategies in terms of metrics such as Sharpe ratio and maximum retracement, the strengths and weaknesses of the dynamic portfolio optimisation model are assessed, and its adaptability and effectiveness in different market environments are analysed. The results are shown below:

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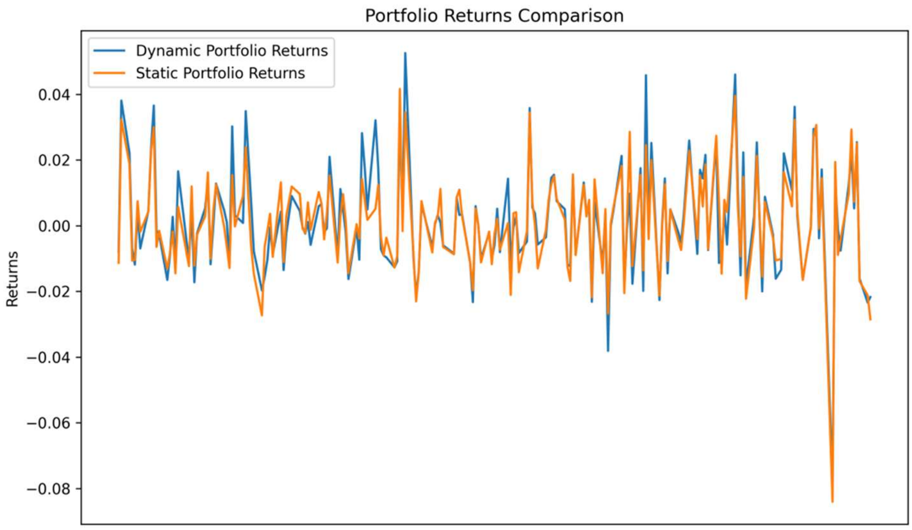

Fig. 9 Comparison of dynamic and static returns

Dynamic portfolio Sharpe ratio: 0.10948675161971819
Static portfolio Sharpe ratio: 0.08518474640838344
Maximum dynamic portfolio retracement: 0.09926058604371413
Maximum static portfolio retracement: 0.10194961279724968

The data showed that dynamic portfolios had higher Sharpe ratios, which implied that they were able to achieve higher excess returns for the same level of risk; alternatively, to achieve the same return, dynamic portfolios took on relatively less risk. On the other hand, the dynamic portfolio had a relatively small maximum retracement, indicating that it was better able to control losses and protect the value of the portfolio during market volatility.

6.4. Robust Optimisation

Further robust optimisation of the dynamic portfolios is carried out to reduce the interference of the parameter estimation errors on the portfolio decisions and improve the robustness of the model by adding robust terms to the objective function, the results are shown below:

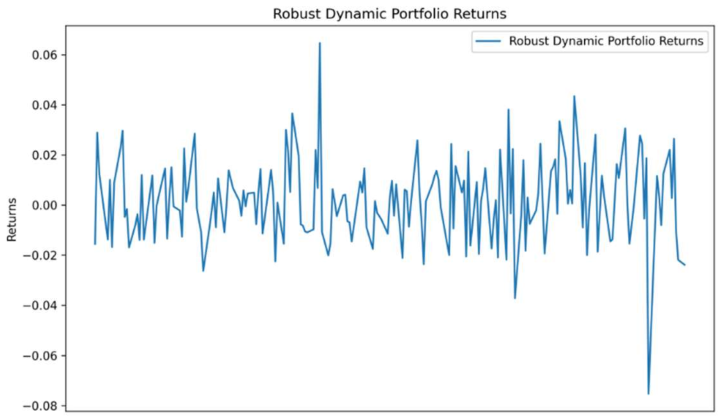

Fig. 10 Robust dynamic portfolio returns

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Robust Dynamic Portfolio Sharpe Ratio: 0.10695881252697699

Robust Dynamic Portfolio Maximum Retraction: 0.08511138257130313

Based on the data, in terms of return-to-risk ratio, the Sharpe ratio of the dynamic portfolio is slightly higher than that of the robust dynamic portfolio, with a numerical difference of 0.0025279390927412, which indicates that the dynamic portfolio can capture slightly more excess returns at the same level of risk. However, this difference is not statistically significant, implying that the two are closer in terms of their ability to capture excess returns per unit of risk . In terms of risk control ability, the Robust Dynamic Portfolio shows a clear advantage, with its maximum retracement reduced by 0.014149203472411, or 14.25%, compared with the Dynamic Portfolio. This indicates that the Robust Dynamic Portfolio is able to control the decline in asset value more effectively in a volatile market environment, significantly reducing the potential loss in extreme market scenarios. This suggests that the robust dynamic portfolio achieves a better trade-off between risk control and return-risk balance. The robust optimisation technique enhances the portfolio's resilience to market uncertainty by introducing an uncertainty ensemble to constrain the parameter fluctuations in the portfolio optimisation model, effectively reducing the portfolio's sensitivity to extreme market scenarios, and thus performing well in controlling the maximum retracement. Although the Robust Dynamic Portfolio is slightly inferior to the Dynamic Portfolio in terms of Sharpe Ratio, its superior performance in risk control makes it a more suitable investment strategy for risk averse investors.

7. Analysis and Discussion of Results

7.1. Validation of Model Validity

The empirical results show that the portfolio model integrating GARCH volatility prediction and dynamic optimisation exhibits better return-risk characteristics than the static portfolio strategy in different market environments. The GARCH model accurately predicts volatility, which provides a reliable risk measure for the dynamic optimisation model; the combination of dynamic and quadratic programming, using robust optimisation to further improve the model performance, enables the portfolio weights to be adjusted in real time according to the market changes, which effectively enhances the adaptability and performance of the portfolio. This verifies the theoretical and practical validity of the model constructed in this study, and provides a more scientific and flexible investment decision-making method for investors .

7.2. Analysis of Impact Factors

The performance of the model is affected by a number of factors: the choice of parameters for the GARCH model is critical to the accuracy of volatility forecasts, and different distributional assumptions (e.g., normal, Student's t-distribution, etc.) can lead to differences in the model's ability to capture the characteristics of market volatility. The rebalancing period setting in the dynamic optimisation model also affects the performance of the portfolio. A shorter rebalancing period can respond to market changes in a more timely manner, but at the same time increase the transaction cost; a longer rebalancing period can reduce the transaction cost, but may miss some market opportunities. In addition, the complexity and uncertainty of the market environment, such as macroeconomic policy adjustments and unforeseen events, can also have an impact on the model's forecasting and optimisation results.

7.3. Comparison with Existing Studies

Compared with existing studies, the innovation of this study lies in the close integration of GARCH volatility prediction and dynamic planning, in addition to the addition of robust optimisation, which constructs a complete dynamic portfolio optimisation system. Most of the existing studies focus on the application of a single model or simply splicing different methods, failing to give full play to the advantages of the synergy of multiple methods. This study not only achieves innovation in methodology, but also uses high-dimensional S&P 500 constituent stock data for validation, which more comprehensively takes into account the complexity of the market and provides new ideas and methods for subsequent research.

8. Conclusion and Outlook

8.1. Conclusion of the Study

This study successfully constructs a portfolio model that combines GARCH volatility prediction and dynamic optimisation, and verifies the effectiveness of the model in improving portfolio return-risk performance through empirical analysis. Under different market environments, the dynamic portfolio strategy has obvious advantages over the static strategy, which can better adapt to market changes and achieve better investment performance for investors. Meanwhile, the study also clarifies the influencing factors of the model and provides directions for further optimisation of the model.

8.2. Inadequate Research

Despite the results achieved in this study, there are still some shortcomings. Firstly, the model assumptions are relatively simplified; in real financial markets, the distribution of asset returns may be more complex, with features such as non-normality and jumps, which may not be fully captured by the model in this study. Second, the study only considered S&P 500 component data, and the applicability of the model to other markets and asset classes requires further validation. In addition, practical factors such as transaction costs and taxes have not been fully considered in the model, which may affect the effectiveness of the model in practical application.

8.3. Future Prospects

Future research is planned to be carried out in the following directions: First, further expand the complexity of the model by introducing more flexible distributional assumptions and more complex volatility models, such as the introduction of the jump diffusion process, in order to better portray the complexity of the financial market characteristics. Second, expand the research scope to multiple markets and different asset classes around the world, and construct a more universal portfolio optimisation model. Third, in-depth study of the impact of transaction costs, taxes and other practical factors on portfolio optimisation, so as to make the model closer to the actual investment scenarios. Combining with machine learning and other emerging technologies, more market information will be explored to enhance the model's prediction and optimisation capabilities, so as to provide investors with more accurate and effective investment decision support.

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