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Differential Game Analysis of Supply Chain Product Innovation: Accounting for Consumer Anticipated Regret and Time-Varying Willingness

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Abstract

The primary objective of this paper is to devise a dynamic differential game model for quality innovation within the realm of technology research and development. This model particularly examines consumer behavior as it is shaped by anticipated regret and time-varying willingness. Our research endeavors to delineate the optimal trajectory of consumers willingness to pay for quality innovations, while concurrently determining the optimal level of quality innovation that ought to be pursued. Moreover, our research undertakes an analysis of differential game equilibrium solutions in the scenario where consumers' anticipated regret is considered in their decision-making processes. This analysis endeavors to illuminate the influence of anticipated regret on strategic decisions pertaining to quality innovation. In addition, numerical simulations are given to verify the obtained theoretical results.

Keywords

Anticipated Regret; Differential Games; Quality Innovation; Product Pricing.

1. Introduction

With the rapid development of science and technology, the survival and growth of enterprises increasingly depend on their ability to innovate and achieve technological breakthroughs [1]. However, despite these efforts, the success rate of innovative products in the market remains a complex issue. Empirical studies have shown that a significant percentage of innovative products fail to succeed in the market, with failure rates ranging from 40% to as high as 90% [2]. The high failure rate of innovative products is frequently attributed to consumers anticipated regret. During the purchasing process, consumers confront uncertainty about the quality and level of innovation of these products. This uncertainty hampers their capacity to anticipate the value that the new product will offer before actually experiencing it. Consequently, consumers engage in anticipatory thinking before making a purchase decision [3].

Anticipated regret has been demonstrated to significantly impact consumer decision-making [4][5]. Although innovative products may promise superior quality, their ultimate influence on the consumer experience remains uncertain. Research consistently reveals that uncertainty-induced regret is a pervasive phenomenon among consumers. This arises as they anticipate potential regret before making a purchase decision. As a result, consumers tend to favor choices that minimize the risk of negative outcomes [6]. For instance, consumers frequently find themselves faced with a choice between two distinct product types: traditional and innovative options. This decision is often complicated by uncertainty [7][8]. In such scenarios, anticipated regret can significantly influence consumers purchasing decisions [9]. Accordingly, it is

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paramount to consider the psychological implications of consumer regret in order to ensure long-term success for innovative products.

Concurrently, the level of quality innovation significantly affects consumers willingness to pay. In practical situations, this willingness is not static but evolves over time. Consumer willingness to pay is defined as the maximum amount of money a consumer is willing to spend to acquire a specific good [10]. As Ang et al. [11] have demonstrated, consumers' anticipated regret and the level of thoughtfulness in their decision-making have been found to positively influence their willingness to pay for their preferred options. In the context of demand-driven pricing strategies, it is crucial for suppliers to gain an understanding of the maximum amount consumers are willing to pay. This understanding is crucial for the pricing decisions of both producers and retailers, who must determine the appropriate price for their products [12][13]. In this study, we aim to conduct an investigation into the state equation that represents consumers willingness to pay for quality innovation. We will employ the level of quality innovation as a key control variable in our analysis. Furthermore, our research will delve into the systematic fluctuations in consumers willingness to pay for quality innovation. Additionally, we will explore the dynamics of the quality innovation level itself, examining how it evolves over time.

In conclusion, this paper incorporates the concept of consumers anticipated regret into the long-term dynamic game between manufacturers of innovative products and those of original ones. Drawing on the utility descriptions of consumers' anticipated regret behavior as presented by Zeelenberg M et al. [5] and Jiang et al. [7], we develop a novel differential game model to address scenario. And we will analyze the optimal strategies for quality innovation within these contrasting frameworks. The culminating segment of our inquiry explores the implications of consumers anticipated regret on the optimal decision-making process pertaining to quality innovation.

In this article, we present several key innovations as follows.

(1) In contrast to the existing literature, which is limited to static models, this paper adopts a novel approach. It meticulously explores the nonlinear dynamics of consumers willingness to pay for quality innovation. Utilizing differential game theory and related methodologies, the study delves into the optimal dynamic trajectory of three pivotal variables: consumers' willingness to pay for quality innovation, the extent of quality innovation, and the pricing strategy.

(2) Drawing upon the stability theory of nonlinear systems, this study delves deeper into examining the stability of equilibrium solutions within the differential game framework.

(3) This study ultimately delves into the repercussions of anticipated regret behavior among consumers on several pivotal aspects. Firstly, it scrutinizes the influence on pricing strategies and profitability margins of both innovative and established products. Secondly, it evaluates the extent to which this behavior impacts the level of quality innovation. Lastly, it assesses the degree to which consumers willingness to pay for quality innovations is affected by their anticipated regret.

2. Problems and Assumptions

2.1. Problem Description

We consider a dynamic market scenario involving two types of products: original products manufactured by incumbent firms, referred to as the original product manufacturer, and innovative products manufactured by new entrants, known as the innovative product manufacturer. Both products share a base attribute, which is normalized to a quality level of 1. The innovative product introduces a quality innovation that enhances its base attribute,

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thereby endowing it with an additional innovative characteristic. The quality innovation level at any given time t is denoted as $q(t)$. The pricing for these products is as follows: the original product is priced at $p_1(t)$, and the innovative product at $p_2(t)$. The quality innovation level influences consumers' willingness to pay for the innovation, which is represented by $v(t)$ at time t. Furthermore, it is essential to account for the anticipated regret that consumers may experience due to uncertainty regarding product quality. To address this, we employ differential game theory and methodology. We begin with a model that considers consumers' anticipated regret. We then proceed to study the optimal strategies for quality innovation under this decision scenario.

The primary research questions addressed in this study are as follows.

(1) What are the optimal trajectories of consumers' willingness to pay for quality innovation and the level of quality innovation over time?

(2) What are the equilibrium solutions of the differential game model, and what is the stability of these solutions?

(3) How does the anticipated regret behavior of consumers influence product decisions?

2.2. Model Assumptions

Assumption 1. Let us consider the existence of a consumer group in the market, which we standardize to a unit size of 1. The consumers' willingness to pay for product quality innovation is represented by $v(t)$, where $v(t)$ is bounded within the interval $[0,1]$. There is a proportionality relationship between the base quality level and the quality innovation level, which can be represented as $\frac{1}{q(t)}$. Moreover, the consumers' willingness to pay for the basic attribute quality of the product is delineated by the formula $\frac{v(t)}{q(t)}$. The forthcoming section presents the equation of state that models consumers' willingness to pay for quality innovation. Through the characterization of this willingness to pay, we discern the inherent dynamic within the consumers' willingness to pay itself. And then the dynamic is delineated as follows.

(1) Within any given time period, the consumers' willingness to pay for raw materials, intermediate products, and final goods supplied by node firms within the supply chain is not boundless. It possesses a potential limit, often referred to as the maximum willingness to pay. The maximum is denoted by the variable k.

(2) The observed increase in consumers' willingness to pay for quality innovation can be attributed to a significant discrepancy. This discrepancy exists between the actual willingness to pay and the potential maximum willingness to pay value. This gap acts as a driving force, propelling the continued rise in willingness to pay. As long as consumers' willingness to pay for quality innovation has not reached its limit, manufacturers will persist in their pursuit of higher returns. They will employ a variety of marketing tactics to achieve this goal. The ultimate objective is to sustain consumers' willingness to pay for these products.

(3) The rate of growth change of $v(t)$ at time t is positively correlated with the difference between k and $v(t)$. Simultaneously, there is a direct relationship between the level of quality innovation in a product and the consumers' willingness to purchase it. The increase in the product's quality innovation level leads to a higher willingness of consumers to purchase, which in turn elevates the rate of change in consumers' willingness to pay for quality innovation. Consequently, the rate of change in the growth of consumers' willingness to pay for quality innovation at any given moment t is positively correlated with the level of quality innovation.

In summary, the dynamic evolution of $v(t)$ can be mathematically expressed as follows:

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

$$\frac{\dot{v}(t)}{v(t)} = m\left[k - v(t)\right] + q(t), \quad v(0) = v_0, \tag{1}$$

where, the symbol $\dot{v}(t)$ represents the instantaneous rate of change in consumers' willingness to pay for quality innovation. The expression $\frac{\dot{v}(t)}{v(t)}$ indicates the rate of change in the growth of consumers' willingness to pay for quality innovation. This ratio captures the dynamic aspect of how quickly the willingness to pay is evolving relative to its current value. The variable m represents the influence factor of consumers' willingness to pay for quality innovation. This factor captures the extent to which various market and product-related factors affect the consumer's willingness of the innovation. The variable k denotes the maximum willingness of consumers to pay for quality innovation. This value represents the upper limit of what consumers are prepared to pay for the product's innovative qualities. Lastly, v_0 is defined as the initial consumer willingness to pay for quality innovation. This initial value sets the starting point for the analysis of how consumers' willingness to pay may change over time or in response to various stimuli.

Assumption 2. Let the unit manufacturing costs for both the original product manufacturer and the innovative product manufacturer be zero. This assumption simplifies the analysis by focusing on the costs associated with research and development. Furthermore, the innovative product manufacturer is engaged in research and development to enhance the quality of the product. The total cost incurred for this research and development on quality innovation is expressed as $\frac{1}{2}cq(t)^2$. Where $q(t)$ represents the quality level of the product's innovation. This equation indicates that the cost of research and development is directly related to the square of the product's quality level, highlighting the increasing investment required for higher levels of innovation. Finally, c is defined as the cost coefficient, a parameter that scales the overall research and development expenditure in relation to the quality level of the innovation.

Assumption 3. Considering the diversity of consumer preferences between the innovative product and the original product, it is reasonable to expect that the probabilities of purchasing these products will vary. To facilitate our analysis, we assume that the probability of a consumer choosing to purchase the innovative product is denoted by x. Consequently, the probability of opting for the original product is $1-x$.

In line with Assumptions 1 and 3, the probability that the valuation of the quality of innovation by consumers, denoted as $q(t)v(t)$, is x. Consequently, the expected valuation of the innovative product by consumers is given by the sum of the base valuation and the additional value attributed to the quality innovation. This is calculated as $\frac{v(t)}{q(t)} + xq(t)v(t)$. In contrast, the expected valuation of the original product, which lacks the quality innovation, is simply the base valuation $\frac{v(t)}{q(t)}$.

Assumption 4. Consumers may experience regret behavior regardless of whether they choose to purchase the original product or the innovative product. This regret can be categorized into two distinct types.

(1) The repeat purchase regret which is allowed to be consumer regret from purchasing the original product, expressed by γ_r.

(2) The switching regret which is allowed to be consumer regret from purchasing an innovative product, represented by γ_s.

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

Assumption 5. Innovative product manufacturers are programmed as leaders in the market, while original product manufacturers are regarded as followers.

3. Model with Consumers' Anticipated Regret

The perceived uncertainty about the product will lead to two primary forms of anticipated regret. The first form is repeat purchase regret, indicated by γ_r. This type of regret emerges when consumers choose to purchase the original product, despite the innovative product boasting superior features. On the other hand, switching purchase regret, represented by γ_s. It often occurs when consumers purchase innovative products, only to find that their functionality and overall experience do not meet expectations. These different types of regrets highlight the complex decision-making process in the product selection process, reflecting the tradeoffs consumers must make between the original product and the innovative product. The negative impact of anticipated regret, as explored by Jiang et al. [7], is mathematically represented as: $A.R. = -\gamma \cdot \text{Prob}(u_f > u_c) \cdot (u_f - u_c)$, where u_f represents the utility derived from the consumer's decision to forgo the purchase, u_c represents the utility derived from the consumer's decision to proceed with the purchase, and $\text{Prob}(u_f > u_c)$ represents the probability that the consumer will experience regret subsequent to making the purchase. The expected utility loss incurred by consumers who opt to purchase the original product can be described as follows

$$\left[u_f - u_c\right]^+ = \left[\left(\frac{v(t)}{q(t)} + v(t)q(t) - p_2(t)\right) - \left(\frac{v(t)}{q(t)} - p_1(t)\right)\right]^+ .$$

Similarly, the expected utility loss associated with consumers' decisions to purchase the innovative product is as follows

$$\left[u_f - u_c\right]^+ = \left[\left(\frac{v(t)}{q(t)} - p_1(t)\right) - \left(\frac{v(t)}{q(t)} - p_2(t)\right)\right]^+ ,$$

where $[X]^+ = \max[X, 0]$.

In addition, we only consider the conditions $\left(\frac{v(t)}{q(t)} + v(t)q(t) - p_2(t)\right) - \left(\frac{v(t)}{q(t)} - p_1(t)\right) > 0$ and $\left(\frac{v(t)}{q(t)} - p_1(t)\right) - \left(\frac{v(t)}{q(t)} - p_2(t)\right) > 0$ in the model incorporating consumers'anticipated regret. This ensures that the two products can coexist in the market.

At this point in time, we examine the utility functions of consumers who are considering the purchase of both the original and the innovative product at time t. Let's start with the following utility functions.

$$\begin{cases} U_1^c(t) = \frac{v(t)}{q(t)} - p_1(t) - x\gamma_r\left[\left(\frac{v(t)}{q(t)} + v(t)q(t) - p_2(t)\right) - \left(\frac{v(t)}{q(t)} - p_1(t)\right)\right], \\ U_2^c(t) = \frac{v(t)}{q(t)} + xv(t)q(t) - p_2(t) - (1-x)\gamma_s\left[\left(\frac{v(t)}{q(t)} - p_1(t)\right) - \left(\frac{v(t)}{q(t)} - p_2(t)\right)\right]. \end{cases} \tag{2}$$

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

For convenience, we define $\rho = \frac{1+\gamma_s}{1+\gamma_r}$ as the relative intensity of consumers' anticipated regre. When $\rho < 1$, which implies $\gamma_r > \gamma_s$, it can be concluded that consumers exhibit a heightened sensitivity to the regret associated with purchasing the original product, particularly when they later learn that the innovative product possesses superior innovative quality. Conversely, when $\rho > 1$, indicating $\gamma_r < \gamma_s$, it is inferred that consumers are more attuned to the regret of purchasing the innovative product, especially upon discovering that it offers only average innovative quality. In the case where $\rho = 1$, or equivalently $\gamma_r = \gamma_s$, it can be concluded that consumers are equally sensitive to both types of regret scenarios.

Under the case $U_1^c(t) = U_2^c(t)$, the critical willingness to pay of consumers represented as $v_2(t)$, which takes intoaccount their anticipated regret, is as follows

$$v_2(t) = \frac{p_2(t) - p_1(t)}{xq(t)}(\rho + x - \rho x). \tag{3}$$

As can be seen from Figure 1, consumer purchasing behavior is shaped by the critical willingness to pay value in relation to their perceived utility. When the value of $v(t)$ is within the range $0 \le v(t) \le v_2(t)$, the utility of theoriginal product exceeds that of the innovative product, such that $U_1^c(t) \ge U_2^c(t)$. Under these conditions, consumers will select the original product. On the contrary, when $v(t)$ exceeds $v_2(t)$ but remains within the interval $v_2(t) < v(t) \le 1$, the utility of the innovative product surpasses that of the original product, indicated by $U_2^c(t) > U_1^c(t)$. In this case, consumers will opt for the innovative product.

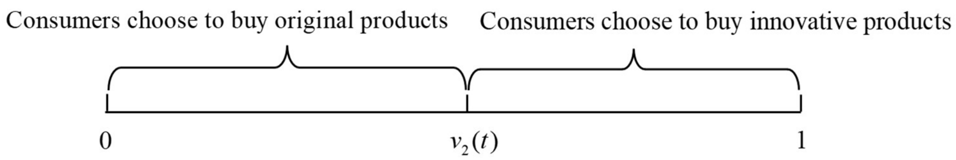

Figure 1. Consumers' willingness to pay for products under consideration of anticipated regret

The demand for a product is represented by the ratio of consumers' willingness to pay interval for that product to the total willingness to pay interval. Therefore, when considering the anticipated regret of consumers, the demand for the original and innovative products can be expressed as follows

$$\begin{cases} D_1^c(t) = \frac{p_2(t) - p_1(t)}{xq(t)}(\rho + x - \rho x), \\ D_2^c(t) = 1 - \frac{p_2(t) - p_1(t)}{xq(t)}(\rho + x - \rho x). \end{cases} \tag{4}$$

Consequently, considering the anticipated regret of consumers, we can derive the instantaneous profit functions for both the original product manufacturer and the innovative product manufacturer. These functions are delineated as follows

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

$$\pi_1^c(t) = p_1(t) \cdot \frac{p_2(t) - p_1(t)}{xq(t)}(\rho + x - \rho x), \tag{5}$$

$$\pi_2^c(t) = p_2(t) \cdot \left[1 - \frac{p_2(t) - p_1(t)}{xq(t)}(\rho + x - \rho x)\right] - \frac{cq(t)^2}{2}. \tag{6}$$

As the manufacturer of the innovative product assumes the role of the leader within the supply chain in this market, it is imperative that they anticipate the price of the original product prior to determining the pricing and quality innovation strategies for their own product. Consequently, the price of the original product must be established first. To determine this price, we first calculate the first-order partial derivatives of the original product price p_1 with respect to the Equation (5). So we can derive the price as follows

$$p_1(t) = \frac{p_2(t)}{2}. \tag{7}$$

Accounting for the anticipated regret of consumers, the innovative product manufacturer aims to identify the optimal quality innovation level and pricing strategy for the innovative product over a continuous time period. The objective is to maximize the discounted value of profits. To achieve this, we substitute the Equation (7) into the Equation (6) and integrate the result with the dynamic equation (1). This process allows us to derive the objective function for the innovative product, which is presented below

$$\begin{aligned} &\max_{p_2, q} \int_0^{+\infty} e^{-rt} \left\{ p_2(t) \cdot \left[1 - \frac{p_2(t)}{2xq(t)}(\rho + x - \rho x)\right] - \frac{cq(t)^2}{2} \right\} dt, \\ s.t. \quad &\dot{v}(t) = mv(t)[k - v(t)] + v(t)q(t), \quad v(0) = v_0, \end{aligned} \tag{8}$$

where r denotes the discount rate.

Let us define the present-value Hamiltonian associated with the objective function (8). It is presented as below

$$H_2(t) = p_2(t) \cdot \left[1 - \frac{p_2(t)}{2xq(t)}(\rho + x - \rho x)\right] - \frac{cq(t)^2}{2} + \lambda_2(t)\left[mv(t)[k - v(t)] + v(t)q(t)\right]. \tag{9}$$

Consequently, the Equation (8) representing the objective function is reformulated as follows

$$\max_{p_2, q} \int_0^{+\infty} e^{-rt} \left[H_2(t) - \lambda_2(t)\dot{v}(t)\right] dt, \tag{10}$$

where $\lambda_2(t)$ is a continuously differentiable Lagrange multiplier function. It satisfies the following condition that $\frac{\partial H_2(t)}{\partial \lambda_2(t)} = \dot{v}(t)$.

The first order condition for the maximization of the present value Hamiltonian function with respect to p_2 is given by

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

$$\frac{\partial H_2(t)}{\partial p_2(t)} = 1 - \frac{p_2(t)(\rho + x - \rho x)}{q(t)x} = 0. \tag{11}$$

The Equation (11) presents the first-order condition, which allows us to derive the price of the innovative product. The derivation is as follows

$$p_2(t) = \frac{x}{(\rho + x - \rho x)} q(t). \tag{12}$$

Based to the Equation (7), we are able to ascertain the pricing of the original product. The calculation is detailed below

$$p_1(t) = \frac{x}{2(\rho + x - \rho x)} q(t). \tag{13}$$

By substituting the Equation (12) into the Equation (9), we can derive the maximizing first-order condition for the present-value Hamiltonian function with respect to $q(t)$. This process also yields the cohomology equation. The detailed procedure and the resulting expressions are presented below

$$\frac{\partial H_2(t)}{\partial q(t)} = v(t)\lambda_2(t) - cq(t) + \frac{x}{2(\rho + x - \rho x)} = 0, \tag{14}$$

$$\dot{\lambda}_2(t) = \lambda_2(t)\left[r - q(t) - km + 2mv(t)\right]. \tag{15}$$

The next step is to examine the change rule of the quality innovation level $q(t)$, taking into account the anticipated regret of consumers. Let's start to derive the first-order condition Equation (14) with respect to t. So we integrate the co-state equation (Equation (15)) into the derived expression. At that point, we are able to deduce the change rule for the quality innovation level $q(t)$. The detailed outcomes are presented as follows

$$\dot{q}(t) = \frac{1}{c}\left[cq(t) - \frac{x}{2(\rho + x - \rho x)}\right]\left[r + mv(t)\right]. \tag{16}$$

These Equations (16) and (1) can be interconnected to form a dynamic control system that incorporates the anticipated regret of consumers. The resulting system is described as follows

$$\begin{cases} \dot{q}(t) = \frac{1}{c}\left[cq(t) - \frac{x}{2(\rho + x - \rho x)}\right]\left[r + mv(t)\right], \\ \dot{v}(t) = mv(t)[k - v(t)] + v(t)q(t), \\ v(0) = v_0, \\ q(0) = q_0, \end{cases} \tag{17}$$

where q_0 is the initial quality innovation level.

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

By employing the method of separation of variables, we have solved the aforementioned dynamic control system to obtain the optimal trajectories for both consumers' willingness to pay for quality innovation and the level of quality innovation. These results take into account the consumers' anticipated regret, as detailed in Proposition 1.

Proposition 1 Under the condition of considering consumers' anticipated regret behavior, we can derive the following results.

(1) The optimal dynamic trajectory of consumers' willingness to pay for quality innovation ischaracterized by

$$v(t)^{c*}=\frac{2c(\rho+x-\rho x)mk+2c(\rho+x-\rho x)e^{A_2+\int r+mv(t)^{c*}dt}+x}{2cm(\rho+x-\rho x)[e^{mkt+\frac{2c(\rho+x-\rho x)e^{A_2+\int r+mv(t)^{c*}dt}+x}{2c(\rho+x-\rho x)}t}+A_1]^{-1}+2cm(\rho+x-\rho x)},$$

where $A_1=\frac{v_0 m}{mk+q_0-v_0 m}-1,\ \ A_2=\ln\left[\frac{2c(\rho+x-\rho x)q_0-x}{2c(\rho+x-\rho x)}\right].$

(2) The optimal dynamic trajectory for the quality innovation level is delineated as follows

$$q(t)^{c*}=\frac{2c(\rho+x-\rho x)e^{A_2+\int r+mv(t)^{c*}dt}+x}{2c(\rho+x-\rho x)}.$$

(3) The optimal dynamic trajectory for the pricing of innovative products can be described asfollows

$$p_2(t)^{c*}=\frac{2cx(\rho+x-\rho x)e^{A_2+\int r+mv(t)^{c*}dt}+x^2}{2c(\rho+x-\rho x)^2}.$$

According to Proposition 1, consumers' anticipated regret can significantly influence both their willingness to pay for quality innovation and the degree of quality innovation achieved. This proposition provides the valuable guidance on assessing consumers' willingness to pay for quality innovation at particular moments, taking into account their potential regret. Furthermore, it offers recommendations for manufacturers of innovative products on how to develop quality innovation strategies during these critical times.

Conclusion 2 By the same token, when incorporating consumers' anticipated regret into the analysis, the optimal dynamic trajectory for the pricing of the original product is stated asfollows

$$p_1(t)^{c*}=\frac{2cx(\rho+x-\rho x)e^{A_2+\int r+mv(t)^{c*}dt}+x^2}{4c(\rho+x-\rho x)^2}.$$

Conclusion 3 According to Proposition 1 and Conclusion 2, we can obtain

$$\frac{\partial v(t)}{\partial\rho}<0,\ \ \frac{\partial q(t)}{\partial\rho}<0,\ \ \frac{\partial p_2(t)}{\partial\rho}<0,\ \ \frac{\partial p_1(t)}{\partial\rho}<0.$$

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

According to Conclusion 3, it follows that consumers willingness to pay for quality innovation, the level of quality innovation, and the price of innovative and original products decrease as the relative intensity of consumers' anticipated regret increases. With heightened relative intensity of regret, consumers become increasingly sensitive to the potential regret associated with purchasing the innovative product. Consequently, their willingness to pay for quality innovation diminishes. In response, the innovative product manufacturer may lower the product's price to attract consumers, especially when they detect a reduction in purchasing motivation. Similarly, the original product manufacturer may reduce the product's price to preserve its competitive edge.

The forthcoming work delves into the equilibrium solutions of differential games, taking into account the behavior of consumers' anticipated regret.

Proposition 4 When the dynamic control system (17) reaches a stable state, we obtain that $\dot{q}(t)=\dot{v}(t)=0$ .Then we can derive the equilibrium solutions. These solutions pertain to consumers' willingness to pay for quality innovation, the level of quality innovation, as well as the prices and profits for both innovative and original products. The work incorporates the consideration of consumers' anticipated regret. These results are presented as follows

$$v^c=\frac{x}{2cm(\rho+x-\rho x)}+k,\ \ q^c=\frac{x}{2c(\rho+x-\rho x)},$$
$$p_2^c=\frac{x^2}{2c(\rho+x-\rho x)^2},\ \ p_1^c=\frac{x^2}{4c(\rho+x-\rho x)^2},$$
$$\pi_2^c=\frac{x^2}{8c(\rho+x-\rho x)^2},\ \ \pi_1^c=\frac{x^2}{8c(\rho+x-\rho x)^2}.$$

Proposition 4 indicates that, in scenarios where consumers' anticipated regret is taken into account, the equilibrium solutions for the profits of innovative and original products mirror each other. This outcome is attributed to the fact that the manufacturer of the innovative product incurs quality innovation research and development costs. Consequently, even though the price of the innovative product is higher, the equilibrium profit of the innovative product is consistent with the equilibrium profit of the original product.

Conclusion 5 The equilibrium solution $\{q^c,v^c\}$ is the saddle point equilibrium.

Proof: According to the stability theory of nonlinear systems as referenced in [14], the Jacobimatrix of the dynamic control system (17) is denoted by $J^c=\frac{\partial(\dot{q}(t),\dot{v}(t))}{\partial(q(t),v(t))}$. At the equilibriumsolution $\{q^c,v^c\}$, the Jacobi matrix is given by

$$J^c=\begin{bmatrix} r+\frac{x}{2c(\rho+x-\rho x)}+mk & 0 \\ \frac{x}{2cm(\rho+x-\rho x)}+k & -mk-\frac{x}{2c(\rho+x-\rho x)} \end{bmatrix}.$$

Let E be the second-order unit matrix and $y_i^c\,(i=1,2)$ be the eigenvalues corresponding to the Jacobian matrix J^c . Calculating the characteristic equation $\left|J^c-y_i^cE\right|=0$, we can obtain the characteristic value as follows

ISSN: 2692-7608 DOI: 10.6981/FEM.202507_6(7).0007

$$y_1^c = r + \frac{x}{2c(\rho + x - \rho x)} + mk > 0, \quad y_2^c = -mk - \frac{x}{2c(\rho + x - \rho x)} < 0.$$

This implies that the eigenvalues consistently exhibit one positive and one negative value, regardless of the variation in the relative intensity of consumers' anticipated regret or adjustments in other variables. Consequently, according to the theory of system stability, we can determine the equilibrium solution $\{q^c, v^c\}$ is a saddle point equilibrium.

In terms of Conclusion 5, we find out that altering strategies by the innovative or the original product manufacturers does not yield additional benefits around the equilibrium state. This is true regardless of changes in the intensity of consumers' anticipated regret or adjustments in their maximum willingness to pay for quality innovation. It indicates that the optimal strategy for both innovative and original product manufacturers subject to consumers' anticipated regret remains to maintain the status quo rather than pursuing strategic changes.

4. Example Analysis

In this section, we delve into an analysis of the influence that the relative intensity of consumers anticipated regret, denoted by ρ, exerts on the decision-making processes of product manufacturers. This investigation leverages numerical simulation to provide insights. The basicparameter values utilized in the simulation are presented in Table 1.

Table 1. Parameter settings

Parameter	m	k	x	c	v_0	q_0	r
Numeric value	2	0.8	0.6	2	0.1	2	0.8

As shown in Table 1, the quality innovation level converges to an equilibrium state at $q^c = \frac{0.6}{4(0.4\rho + 0.6)}$ a specific value of the relative intensity of anticipated regret (ρ). The optimal trajectoryof consumers willingness to pay for quality innovation, in relation to the relative intensity of their anticipated regret (ρ), is depicted in Figure 2.

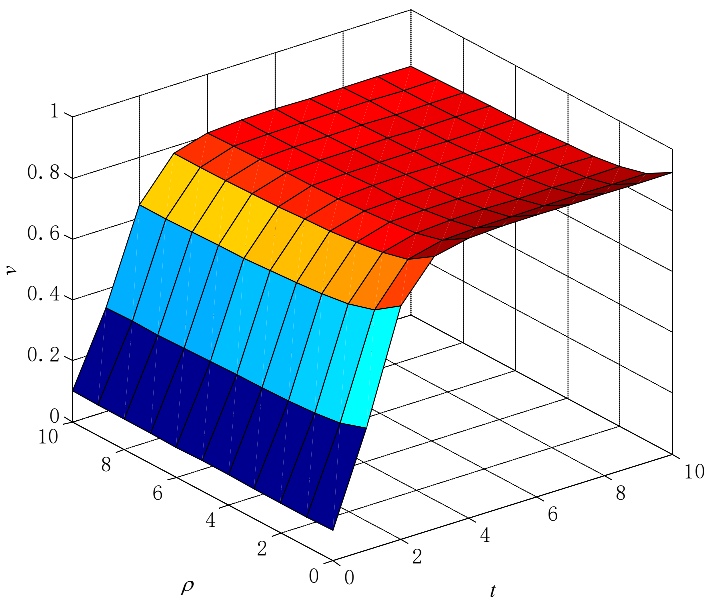

Figure 2. The impact of ρ and t on consumers' willingness to pay for quality innovation

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We explore the combined influence of anticipated regret intensity and the passage of time on consumers willingness to pay for quality innovation in Figure 2. This study is conducted under the premise that the quality innovation level has attained an equilibrium state, specifically at a particular relative intensity of anticipated regret. As illustrated in Figure 3, we discern the following trends.

(1) As time passes, consumers willingness to pay for quality innovation tends to rise, with this trend persisting until it plateaus at an equilibrium point.

(2) As the intensity of anticipated regret escalates, the willingness to pay for such innovation correspondingly diminishes, ultimately reaching a specific level.

This is because, at a specific equilibrium point of quality innovation (i.e., when quality remains stable), consumers willingness to pay for these innovative quality improvements tends to increase over time, owing to their growing appeal. However, this upward trend is constrained and ultimately plateaus at a maximum level of willingness to pay. An increase in the relative intensity of consumers anticipated regret, which indicates a greater sensitivity to the potential regret associated with purchasing innovative products, leads to a decrease in the equilibrium value of quality innovation. Simultaneously, the optimal willingness to pay for quality innovation also reduces. When the relative intensity of consumers anticipated regret is higher, the equilibrium value of the quality innovation level is lower. In these circumstances, consumers willingness to pay for quality innovation tends to stabilize at a relatively consistent level.

Therefore, innovative product manufacturers should strive to mitigate the relative intensity of consumers anticipated regret. By doing so, they can enhance consumers purchasing intentions, ultimately leading to increased returns.

According to the parameter settings outlined in Table 1, a distinct value of anticipated regret is identified at which point consumers willingness to pay for quality innovation attains a stable equilibrium. This equilibrium is mathematically formulated as $v^c = \frac{0.6}{8(0.4\rho + 0.6)} + 0.8$.

Moreover, we illustrate the interrelationship between the optimal trajectories of three key variables-namely, the level of quality innovation, the price of the innovative product, and the price of the original product-and the intensity of consumers anticipated regret in Figures 3 and 4.

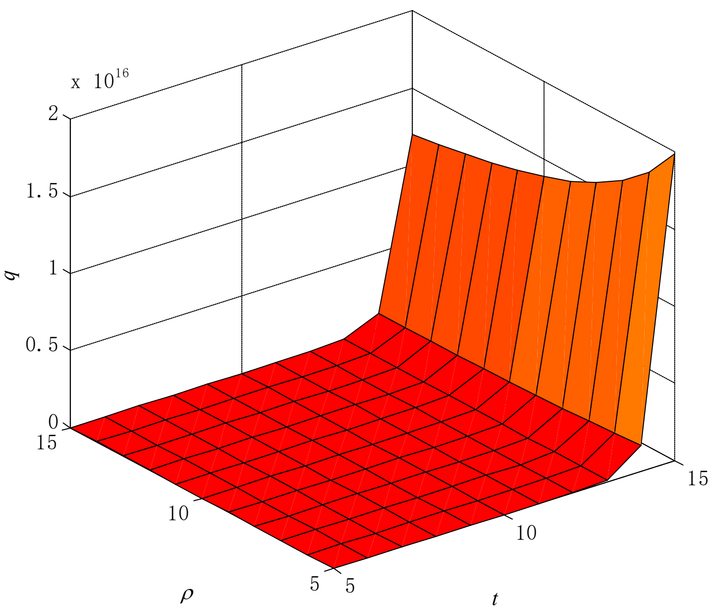

Figure 3. The impact of ρ and t on the quality innovation level

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Now we explore the combined effects of consumers anticipated regret intensity and time on the level of quality innovation in Figure 3. This analysis is conducted under the condition where consumers willingness to pay for quality innovation is at an equilibrium state for a given relative intensity of anticipated regret. As depicted in Figure 4, we can draw the following conclusions.

(1) The level of quality innovation gradually increases over time, initially at a moderate pace, which subsequently accelerates.

(2) Conversely, as the relative intensity of consumers anticipated regret intensifies, the level of quality innovation diminishes, eventually plateauing at a stable level where further change ceases.

This phenomenon arises because, when consumers willingness to pay for quality innovation remains at a particular equilibrium, innovative product manufacturers must strive to enhance the level of quality innovation in their offerings in order to attain greater profits. As a result, the level of quality innovation generally tends to escalate over time. However, as the relative intensity of consumers anticipated regret intensifies, the equilibrium level of their willingness to pay for quality innovation diminishes, thereby eroding the incentive for innovative product manufacturers to enhance quality. Consequently, as the relative intensity of consumers anticipated regret escalates, the level of quality innovation declines. When the equilibrium value of consumers willingness to pay for quality innovation is lower as a result of a higher relative intensity of consumers anticipated regret, the level of quality innovation tends to remain relatively stable.

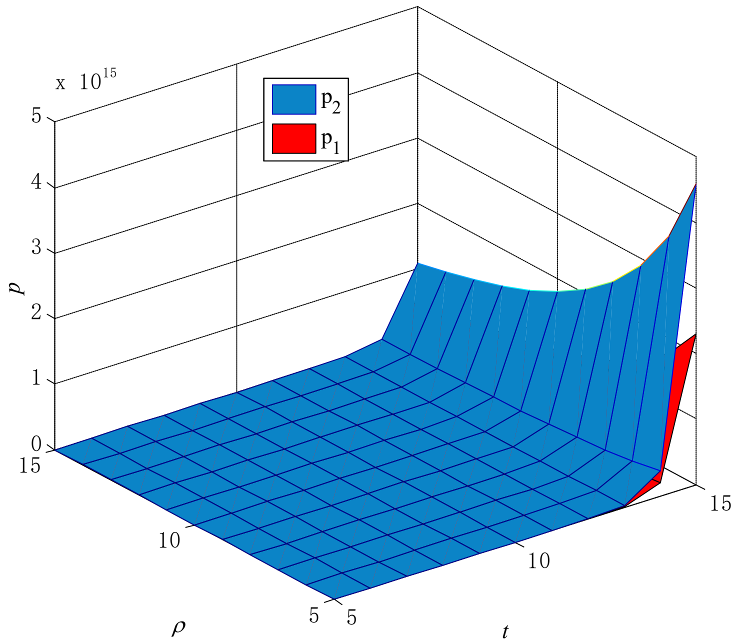

Figure 4. The impact of ρ and t on the price of innovative and original products

Next, we investigate the combined effects of the relative intensity of consumers anticipated regret and the passage of time on the pricing of innovative and original products, as illustrated in Figure 4. This analysis is undertaken under the assumption that consumers willingness to pay for quality innovation is at equilibrium for a given level of anticipated regret. Upon careful examination of Figure 4, we discern the subsequent results.

(1) Over time, the prices of both innovative and original products tend to escalate, with an initial moderate pace of growth. However, as time progresses, the rate of this increase accelerates notably.

(2) Conversely, a heightened relative intensity of consumers anticipated regret triggers a decline in the prices of both product types. Eventually, these prices plateau at a relatively stable level.

Since when consumers' willingness to pay for quality innovation reaches a stable equilibrium, the innovative products manufacturer aims to maximize their profits by incrementally raising the prices of their products. Similarly, the original product manufacturer tends to adopt the same strategy, increasing their prices to remain competitive. During the initial stages of product sales, in order to attract consumers, manufacturers set a moderate pace for price increases. However, as the product gains market share, manufacturers confront a distinct challenge: to sustain or enhance profitability, they must accelerate the rate of price increases. As the relative intensity of consumers anticipated regret intensifies, their willingness to pay for innovative products dwindles. Consequently, this diminished willingness to pay prompts innovative product manufacturers to reduce their prices, thereby presenting more attractive options and enhancing the utility and appeal of their products to consumers.

5. Conclusion

Our study offers a comprehensive analysis of several key factors, such as consumer anticipated regret, the willingness of consumers to pay for quality innovation, the level of quality innovation, and the long-term strategic interactions between manufacturers of innovative products and those of original ones. We construct a dynamic differential game model centered around the development of quality innovation technology. This model is designed to explore the optimal dynamic trajectories of three critical variables: consumers willingness to pay for quality innovation, the level of quality innovation achieved and the prices of the products.

Our research has pinpointed several pivotal areas for future enhancement and optimization.

(1) Constant Influence Coefficient Assumption. The model, which represents the equation of state for consumers willingness to pay for quality innovations, presupposes a constant influence coefficient, denoted as m. This coefficient, primarily influenced by a confluence of factors including cost, scale, and product quality within the firm, may not remain immutable and could potentially evolve over time. Therefore, future research endeavors should contemplate the temporal dynamics of this coefficient in order to bolster the models predictive prowess.

(2) Limited Control Variable Scope. The present model predominantly centers on the quality innovation level as the sole control variable. Nevertheless, consumers readiness to pay for quality innovations is likely to be influenced by a wider array of variables, encompassing not only variations in the quality innovation level but also fluctuations in product pricing.

Incorporating these additional factors into future models would offer a more comprehensive comprehension of consumer behavior and unlock novel avenues for research.

Availability of Data Andmaterials

All of the authors declare that all the data can be accessed in our manuscript in the numerical simulation section.

Conflicts of Interest

The authors declare that there is no conflict of interests.

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