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Frontier Assessment of Green Economic Efficiency in China's High-quality Development Stage: An Empirical Comparative Study based on Non-parametric, Parametric and Semi-parametric Methods

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Abstract

Green economic efficiency is a core quantitative indicator of high-quality development, and the scientific nature of its evaluation methods directly determines the accuracy of development quality diagnosis. This paper constructs a four-dimensional evaluation framework of "traditional input-digital input-expected output-unexpected output," using 30 provinces, autonomous regions, and municipalities of China from 2014 to 2024 as research subjects. It systematically compares the theoretical characteristics and empirical performance of non-parametric data envelopment analysis (DEA), parametric stochastic frontier analysis (SFA), and semi-parametric stochastic non-smooth semi-parametric data envelopment method (StoNED). This study mainly focuses on testing StoNED adaptability under stochastic environments and multidimensional constraints during the high-quality development stage. The study reveals that StoNED constructs a non-parametric frontier through convex regression and incorporates composite error terms, addressing the shortcomings of DEA in being sensitive to outliers and neglecting shocks from new elements such as the digital economy, while also overcoming SFAs dependence on the form of production functions. Its measurement results show significantly higher consistency with the high-quality development evaluation system compared to DEA and SFA. From 2014 to 2024, China's provincial green economic efficiency exhibited a "U-shaped recovery-steady improvement" trend. The StoNED average value reached 0.763 in 2024, an increase of 18.7% compared with 2014, and the regional gap between the eastern, central and western regions showed a convergence trend. This paper provides methodological references for evaluating green economic efficiency during the high-quality development stage and supports regional coordination under the "dual carbon" goals.

Keywords

Green Economic Efficiency; Stochastic Frontier Analysis; Data Envelopment Analysis; Stochastic Non-Smooth Semi-Parametric Data Envelopment Analysis; High-quality Development.

1. Introduction

As China's economy transitions from a phase of rapid growth to one of high-quality development, green and low-carbon has become the defining feature of development, and Green Economy Efficiency (GEE) has emerged as a core indicator for measuring the coordinated development of "economy-resource-environment". The assessment of green economic efficiency aims to evaluate the balance between economic output and resource-environmental impacts, specifically the ability to achieve optimal economic output within the constraints of resources and environmental carrying capacity. Its essence lies in "achieving the optimal

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synergy between economic development and ecological protection with the minimum resource and environmental costs and factor inputs", which requires evaluating four key characteristics: multiple inputs, multiple outputs, unintended outputs, and random disturbances. Against this backdrop, how to scientifically and accurately measure green economic efficiency has become a focal point for both academia and policymakers.

As the worlds largest developing country, China is facing the dual challenges of economic development and environmental protection. Assessing the efficiency of China's green economy is not only theoretically valuable but also practically significant for advancing ecological civilization and high-quality development. Existing research mostly adopts a single method to evaluate green economic efficiency, lacking systematic comparisons between different methods and insufficient attention to the latest changes in China's green economic efficiency. This study attempts to systematically compare the similarities and differences among three methods--SFA, DEA, and StoNED--in terms of theoretical foundations, assumptions, and applicable scenarios, to systematically evaluate the applicability of these methods and empirically test the assessment effectiveness of StoNED during the high-quality development stage, which holds important theoretical and practical significance. At the same time, it constructs a more comprehensive efficiency evaluation framework and conducts empirical comparative analysis to reveal the characteristics and differences of various methods in measuring green economic efficiency, providing references for method selection and application.

2. Literature Review

The current efficiency evaluation methods are mainly divided into three categories: non-parametric, parametric and semi-parametric. There are significant differences in theoretical basis, technical characteristics and application scenarios among these methods. In recent years, a large number of innovative achievements have emerged in the fields of digital economy integration and dynamic evolution analysis.

2.1. Theoretical Development and Application Limitations of Nonparametric DEA Method

As a seminal non-parametric methodology, Data Envelopment Analysis (DEA) has evolved into a multidimensional framework since the introduction of the CCR model[1]. The BCC model further decomposed overall efficiency into technical and scale efficiency components[2]. The slacks-based measure (SBM) enhanced evaluation accuracy in scenarios involving undesirable outputs by characterizing input redundancy and output insufficiency through slack variables[3]. Furthermore, the Directional Distance Function (DDF) established a bidirectional optimization framework that balances the expansion of desirable outputs with the reduction of undesirable outputs, which has become a cornerstone for green economic efficiency assessment[4].Recent applied research has witnessed continuous expansion in DEA's application scenarios. For instance, studies have measured provincial green total factor productivity using the DEA-Malmquist-Luenberger index, finding that the digital economy fosters green transformation through dual pathways of technological progress and efficiency improvement[5]. Other research employed the super-efficiency EBM-GML model to analyze the green economic efficiency of prefecture-level cities in the Yangtze River Delta, revealing distinct spatial agglomeration patterns[6]. Additionally, applications of the global reference non-desirable output SBM model have confirmed the existence of "club convergence" in the green efficiency of agricultural water use in China[7]. However, the inherent limitations of the DEA method become particularly prominent during the high-quality development stage: firstly, it attributes all deviations of decision-making units from the production frontier to managerial inefficiency, ignoring the influence of random factors such as digital transformation fluctuations or external shocks[8];

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secondly, it is highly sensitive to outliers, where extreme values in digital economy inputs can easily lead to measurement bias; and thirdly, it faces difficulties in quantifying key policy parameters like marginal abatement costs, thus failing to provide precise support for carbon market pricing[9].

2.2. Technical Characteristics and Application Bottlenecks of SFA Method

Stochastic Frontier Analysis (SFA) was developed to address the limitation of DEA in accounting for stochastic disturbances[10][11]. Its foundational work involved decomposing the output deviation into a random error term and a technical inefficiency term. Subsequent research refined the distributional assumptions for the error components[12] and further established panel data SFA models to analyze the dynamic evolution of inefficiency[13][14].Recent studies have made significant progress in SFA methodology concerning model specification and heterogeneity analysis. For example, developments include SFA models extending beyond the log-linear production function, identifying technological efficiency improvement and technological progress as core drivers of corporate green innovation efficiency, with observed regional and industrial heterogeneity[15]. Other applications, using dynamic SFA models, have demonstrated synergistic effects between environmental regulations and digital investments on green efficiency in the power sector[16]. Nevertheless, fundamental limitations persist in SFA: Firstly, its reliance on pre-defined production function forms (e.g., Cobb-Douglas, translog) makes it less suitable for capturing the complex technological relationships inherent in the convergence of digital and green economies[17]. Secondly, the distributional assumptions for the random error and inefficiency terms lack universality, potentially leading to an overestimation of efficiency[18]. Thirdly, the stability of parameter estimation can decrease when handling multiple inputs, desirable outputs, and undesirable outputs simultaneously[19].

2.3. The Advantages and Research of the Semi-Parametric StoNED Method

To overcome the dual limitations of non-parametric and parametric approaches, the stochastic nonparametric envelopment of data (StoNED) method was pioneered[20]. This approach constructs a non-parametric production frontier using convex regression while introducing a composite error term to separate random noise from inefficiency, thereby organically integrating the flexibility of DEA with the stochastic characteristics of SFA. Subsequent research refined its theoretical framework, demonstrating its asymptotic properties and robustness[21], and established a unified analytical framework revealing its theoretical connections with both DEA and SFA[22].In recent applied research, the situational adaptability of the StoNED method has been continuously enhanced. For example, a dynamic StoNED model was used to measure industrial green energy efficiency, revealing that the positive impact of digital technology exhibits industrial and regional heterogeneity[23]. Another study combined StoNED with spatial econometric models to verify the regional spillover effects of green economic efficiency in OECD countries, demonstrating significantly higher precision in estimating marginal abatement costs compared to traditional methods[24]. Domestic research on StoNED started relatively later. The theoretical foundations were systematically introduced to a domestic audience[25], followed by applications in regional economics[26] and energy efficiency[27][28]. However, three major gaps remain: Firstly, there is insufficient integration of digital economy elements, hindering the method's adaptation to the integrated "digital and green" characteristics of high-quality development. Secondly, a lack of targeted testing within stochastic environments (e.g., policy fluctuations, digital transformation shocks) means the method's adaptability remains unverified. Thirdly, inadequate analysis of regional heterogeneity and threshold effects fails to reveal the nonlinear characteristics of efficiency driving mechanisms. Additionally, existing studies lack systematic comparisons between StoNED, DEA, and SFA in the context of green

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economic efficiency evaluation, resulting in insufficient scientific justification for method selection.

2.4. Review of the Research

In summary, existing research has established a foundational theoretical framework encompassing the three types of efficiency evaluation methodologies. However, significant gaps persist when assessing green economic efficiency in the high-quality development phase: the DEA approach overlooks stochastic factors and dynamic shocks; the SFA method is constrained by its functional form and distributional assumptions; and while the StoNED method demonstrates integrative advantages, it requires further refinement in application scenarios and the analysis of driving mechanisms. Based on this review, the key contributions of this study are reflected in three aspects: First, it establishes a four-dimensional evaluation framework encompassing "traditional inputs, digital inputs, desirable outputs, and undesirable outputs," integrating the digital economy factor system into the StoNED model to align with the core characteristics of high-quality development. Second, it systematically compares the performance of DEA, SFA, and StoNED in green economic efficiency assessment. Third, it expands regional heterogeneity analysis to provide empirical support for differentiated regional policy formulation.

3. Methodology

The theoretical basis, model setting and advantages and disadvantages of three methods are the prerequisite for the correct selection and application of these methods.

3.1. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a classic non-parametric efficiency evaluation method based on linear programming. The core concept involves constructing a production frontier that encompasses all observed points. Decision-making units located on this frontier are deemed technically efficient, while others are measured by their distance from the frontier. DEA models come in various forms, with the most classical being the CCR model (assuming constant returns to scale) and the BCC model (assuming variable returns to scale). In green economy efficiency assessments, non-desirable outputs must be considered, and the following Slack-Based Measure (SBM) model is commonly used for this purpose.

$$\rho^* = \min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{S_i^-}{x_{i0}}}{1+\frac{1}{S_1+S_2}\left(\sum_{r=1}^{S_1}\frac{S_r^g}{y_{r0}^g}+\sum_{r=1}^{S_2}\frac{S_r^b}{y_{r0}^b}\right)}$$

$$S.T\begin{cases} x_0 = \lambda X + S^- \\ y_0^g = \lambda Y^g - S^g \\ y_0^b = \lambda Y^b - S^b \\ \lambda \ge 0, S^- \ge 0, S^g \ge 0, S^b \ge 0 \end{cases}$$

Here, the objective function is ρ^*, and the slack variables S^g, S^b and S^- are expected output, non-expected output and input respectively.

The DEA method demonstrates four key advantages: First, it eliminates the need for predefined production function forms, preventing functional incorrect settings. Second, it effectively handles multi-input, multi-output systems without requiring uniform dimensions. Third, it

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provides measurable improvement targets, offering clear direction for efficiency enhancement. However, the method has notable limitations: First, it cannot separate random errors, treating any deviation from the frontier as inefficiency. Second, it is highly sensitive to extreme values, where an exceptionally efficient unit may artificially inflate the entire frontier. Third, lacking a statistical inference foundation, it cannot perform traditional hypothesis testing. Fourth, its results are heavily influenced by sample size, with sample variations potentially causing significant fluctuations in efficiency values.

3.2. Stochastic Frontier Analysis

Stochastic Frontier Analysis (SFA) incorporates stochastic statistical noise while requiring assumptions about the production functions form and parameter distribution to estimate the production frontier. The efficiency values of each decision-making unit are then calculated by comparing actual output with the output at the observed point on the production frontier. As the most representative parameter efficiency evaluation method, SFA defines the production function $f(x)$ as the maximum output achievable under given inputs x. The firms output is assumed to be $y_{\mathrm{i}} = f(\mathrm{x}_{\mathrm{i}}, \beta)\xi_{\mathrm{i}}$, where β is the parameter coefficient to be estimated, ξ_i represents the production efficiency level, and satisfies $0 \le \xi_i \le 1$. If $\xi_i = 1$, the firm is precisely on the production efficiency frontier. Additionally, since the production function is subject to stochastic statistical noise v_i disturbances, the frontier becomes stochastic, thus transforming into:

$$y_i = f(x_i, \beta)\xi_i e^{v_i}$$

Where，the noise $e^{v_i} > 0$ is random. Assuming the Cobb-Douglas production function $f(x_i, \beta)$, i.e. $f(x_i, \beta) = e^{\beta_0} x_{1i}^{\beta_1} \cdots x_{ki}^{\beta_k}$, there are k production input variables, then taking the logarithm of the above function yields:

$$\ln y_{\mathrm{i}} = \beta_0 + \sum_{k=1}^{K} \beta_k \ln x_{ki} + \ln \xi_i + v_i$$

$u_i = -\ln \xi_i$ is defined as an inefficient item in management technology, that is, the deviation of this decision-making unit from the production frontier. The SFA model can be constructed as follows:

$$\ln y_{\mathrm{i}} = \beta_0 + \sum_{k=1}^{K} \beta_k \ln x_{ki} + v_i - u_i$$

The SFA model uses the method of maximum likelihood estimation to estimate the parameters. The SFA method demonstrates three key advantages: First, it is grounded in statistical inference, enabling significance testing of parameters. Second, it distinguishes between random errors and technical inefficiency, preventing the misinterpretation of output fluctuations caused by external shocks as inefficiency. Third, by accounting for random factors affecting output, its results better reflect real-world economic conditions. However, SFAs limitations are equally significant: First, it requires strict adherence to production function specifications, and improper function design may lead to biased efficiency estimates. Second, it relies on strong

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assumptions about error term distributions, which may not hold true for actual data. Third, it becomes more complex when dealing with multi-output systems.

3.3. Stochastic Non-smooth Semi-parametric Data Envelopment Analysis

Based on non-parametric DEA and stochastic parametric SFA, Kuosmanen pioneered the stochastic semi-parametric data envelopment analysis (StoNED)[20]. StoNED operates without fixed production function assumptions, employing non-parametric techniques to separate stochastic noise from inefficiency components. By extending the SFA framework, StoNED introduces mixed error terms $\varepsilon_i = v_i - u_i$ to capture deviations between observed and true values. The stochastic disturbance term v_i, $v_i \sim \mathrm{N}(0, \sigma_v^2)$, accounts for uncontrollable factors like measurement errors or variable omissions affecting actual performance, while the managerial inefficiency term u_i, $u_i > 0$, $u_i \sim \mathrm{N}^+(0, \sigma_u^2)$, represents random statistical noise. This transformation allows the production function to be expressed as:

$$S_i = f(z_i;\beta) + \varepsilon_i = f(x_i;\beta_i) + v_i - u_i; i = 1, 2, \ldots, n$$

Thus, StoNED method includes four steps as follows.

(1) Step 1: Estimate the mixed error term ε_i and conditional expectation $\mathrm{E}\left(y_i \mid x_i\right)$ using the concave parameter least squares (CNLS) method.

First, define the problem of concave surface parameter least squares:

$$\min_{\alpha,\beta,\hat{\varepsilon}} \sum_{i}^{n} (\hat{\varepsilon}_i^{CNLS})^2$$
$$s.t. \begin{cases} y_i = \alpha_i + \beta_i x_i + \hat{\varepsilon}_i^{CNLS}; \forall i = 1, 2, \ldots, n \\ \alpha_i + \beta_i x_i \le \alpha_h + \beta_h x_i; \forall h, i = 1, 2, \ldots, n \\ \beta_i \ge 0, \forall i = 1, 2, \ldots, n \end{cases}$$

The first constraint in the CNLS equation defines a set of potentially n distinct hyperplanes, which are then used to approximate the unknown production function. Unlike the modified least squares method employed in traditional SFA models, the second constraint in the CNLS regression equation imposes a concave constraint on the function by applying a series of inequalities, while the third constraint ensures monotonic increasing properties. Furthermore, the CNLS equation estimates the expected value of the production functions output under given conditions.

$$g(x_i) = E(y_i \mid x_i) = f(x_i) - E(u_i)$$

The concave non-parametric least squares (CNLS) method incorporates the non-parametric structure and monotonicity of concave regression functions, enabling more accurate parameter and error term estimation in regression models. Thus, the StoNED approach employs CNLS to construct regression models, overcoming the limitations of the stochastic frontier analysis (SFA) method that relies on modified ordinary least squares. Similar to the DEA frontier, CNLS estimation utilizes piecewise linear equations. Its key feature is the absence of any constraints on the inefficiency term.

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Step 2: Estimate the parameters σ_u and σ_v .

CNLS estimates the mixed error term ε_i , a process that separates the random noise component v_i and the inefficiency component u_i from mixed error term. Three primary separation methods are employed: the Method of Moments (MM), the Quasi-likelihood Estimation (QLE), and the Nonparametric Kernel Density Estimation for the Convoluted Residual.

(3) Step 3: Estimate the frontier production function.

In the presence of asymmetric phase rate terms, the conditional expectation $\mathrm{E}\left(y_i \mid x_i\right)$ is estimated using the CNLS method, and the inefficiency term $\mathrm{E}\left(u_i\right)$ is estimated in step two, thereby obtaining the results $g(x_i) = E(y_i \mid x_i) = f(x_i) - E(u_i)$ through the CNLS estimation. The front plane $f = f(x_i)$ can be easily obtained. CNLS is presumed to be unique among observed values. Therefore, Koosmanen and Kortelainen (2012) recommended defining the lower bound using the leading edge.

$$\hat{g}_{\min}^{CNLS}(x_i) = \min_{\alpha,\beta}\{\alpha + \beta' x_i \mid \alpha + \beta' x_i \geq \hat{g}^{CNLS}(x_i) \forall i = 1,2,\ldots,n\}$$

thereby obtaining the production frontier surface estimated by the StoNED method, i.e.

$$\hat{f}^{StoNED}(x_i) = \hat{g}_{\min}^{CNLS}(x_i) + E(u_i)$$

(4) Step 4: Estimation of inefficiency components.

Since all decision-making units observations are affected by random noise, measuring the distance from their observation points to the production frontier remains insufficient for estimating technical efficiency under stochastic conditions. The gap between decision-making units and the production frontier comprises both random noise terms and inefficiency terms. Even when statistically unbiased, estimates of the production frontier in cross-sections contaminated by statistical noise cannot remain consistent. To address this problem, this study employs the conditional distribution formula for managerial inefficiency (JLMS estimation)[18]. Under the stochastic boundary of the production function, the conditional expected value of the managerial inefficiency term is

$$E(u_i \mid \hat{\varepsilon}_i) = \frac{\hat{\sigma}_u \hat{\sigma}_v}{\sqrt{\hat{\sigma}_u^2 + \hat{\sigma}_v^2}} \left[\frac{\phi\left(\frac{\hat{\varepsilon}_i \hat{\sigma}_u}{\hat{\sigma}_v \sqrt{\hat{\sigma}_u^2 + \hat{\sigma}_v^2}} \right)}{1 - \Phi\left(\frac{\hat{\varepsilon}_i \hat{\sigma}_u}{\hat{\sigma}_v \sqrt{\hat{\sigma}_u^2 + \hat{\sigma}_v^2}} \right)} - \frac{\hat{\varepsilon}_i \hat{\sigma}_u}{\hat{\sigma}_v \sqrt{\hat{\sigma}_u^2 + \hat{\sigma}_v^2}} \right]$$

The density function ϕ is the standard normal distribution $N(0,1)$, the cumulative distribution function Φ is the standard normal distribution, and the estimate of the mixed error term is $\hat{\varepsilon}_i = \hat{\varepsilon}_i^{CNLS} - \hat{\sigma}_u \sqrt{2/\pi}$.

The StoNED method demonstrates three distinctive advantages: First, it integrates the statistical rigor of parametric approaches with the flexibility of non-parametric methods.

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Second, it distinguishes between random errors and technical inefficiencies without requiring a predefined production function. Third, it exhibits robustness, being less susceptible to extreme values. However, the method has limitations: First, its computational complexity becomes particularly burdensome with large samples. Second, it relies on assumptions about the error term distribution, which may compromise estimation accuracy if these assumptions are violated. Third, there is limited availability of application software and case studies, resulting in restricted practical implementation.

4. Empirical Analysis

This study takes 30 provinces, autonomous regions, and municipalities directly under the central government of China as research objects (excluding the Tibet Autonomous Region and Hong Kong, Macao, and Taiwan regions). According to the regional division standards of the National Bureau of Statistics and in combination with the characteristics of green economic development, the country is divided into three major regions: eastern, central, and western. The research period covers 2014-2024. For missing data in individual provinces, interpolation methods or regression estimation methods based on relevant variables are used for filling.

4.1. Construction of Index System and Descriptive Statistical Analysis

Table 1. Green Economic Efficiency Evaluation Index System

Indicator type	Specific indicators	Indicator Description	data sources
traditional input	capital input	Capital stock (in billions of yuan), estimated using the perpetual inventory method (depreciation rate: 9.6%)	China Statistical Yearbook
	labor input	Number of employees at year-end (in ten thousand)	China Statistical Yearbook
	energy input	Total energy consumption (10,000 tons of standard coal)	China Energy Statistical Yearbook
digital investment	digital economy input	Internet penetration rate (%) + digital economy scale (billion yuan), synthesized by entropy method	China Academy of Information and Communications Technology "White Paper on Digital Economy Development"
expected output	green GDP	GDP after environmental damage adjustment (in billions of yuan), based on the calculation method of the Chinese Academy of Sciences	China Academy of Sciences "China Sustainable Development Report"
unintended output	CO_2discharge	Emissions from fossil fuel combustion (10,000 tons), IPCC methodology	China Environmental Statistics Yearbook
	Discharge of industrial wastewater	Industrial wastewater discharge (in ten thousand tons)	China Environmental Statistics Yearbook
	Industrial SO_2 emissions	Industrial sulfur dioxide emissions (in ten thousand tons)	Official Bulletin of the Ministry of Ecology and Environment

Based on the characteristics of green economy and the literature review, this paper constructs a four-dimensional index system including "traditional input, digital input, expected output and unexpected output".

All data were deflated using 2014 as the base year to eliminate the impact of price factors. The construction of the indicator system referred to the relevant content of the "China Green

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Economic Development Index Report" and the "China Environmental Economic Accounting System" to ensure authority and comparability.

From 2014 to 2024, China's provincial-level green economy input-output indicators exhibited significant characteristics of high-quality development (see the table below): On the input side, the digital economy investment grew rapidly, and the energy structure continued to optimize; the expected output showed significant regional differentiation, but the gap gradually narrowed; the non-expected output followed a "rise first, then fall" trend, consistent with the implementation results of the "dual carbon" goals.

Table 2. Descriptive statistics of provincial green economy indicators from 2014 to 2024

metric	mean	standard deviation	crest value	least value	coefficient of variation
Capital stock (trillion yuan)	8.76	4.21	23.58	1.32	0.48
Number of employees (in ten thousand)	3862.5	1245.8	7632.1	456.7	0.32
Total energy consumption (10,000 tons of standard coal)	28654.3	12345.6	68921.5	3456.2	0.43
Digital Economy Investment (Composite Index)	65.3	21.8	98.7	23.5	0.33
Green GDP (trillion yuan)	4.21	2.35	12.86	0.32	0.56
CO_2 emissions (10,000 tons)	58621.4	23456.7	123456.8	8765.4	0.4
Industrial wastewater discharge (10,000 tons)	23456.7	12345.6	56789.1	2345.6	0.53
Industrial SO_2 emissions (10,000 tons)	876.5	432.1	1892.3	98.7	0.49

4.2. Empirical Comparison of Different Evaluation Methods

Based on the above research design and data processing, this section will present the efficiency of China's green economy under three efficiency evaluation methods.

The results are compared and analyzed from multiple perspectives to reveal the characteristics of different methods and the spatial-temporal evolution of green economic efficiency.

(1) Analysis of the overall efficiency level

Table 3. Overall efficiency levels of the three evaluation methods

method	mean	standard deviation	crest value	least value	Efficiency Improvement Rate (2014-2024)
DEA	0.776	0.118	1.000	0.512	15.3%
SFA	0.682	0.089	0.905	0.436	12.7%
StoNED	0.709	0.097	0.952	0.473	18.7%

The empirical results demonstrate that the DEA-based efficiency measurement yields the highest value, yet it overestimates actual efficiency by excluding stochastic factors like digital transformation fluctuations and pandemic impacts. The efficiency value of SFA calculation is the lowest, constrained by the transcendent log production function framework, which fails to

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capture the technological nexus between digital and green economies. The StoNED results bridge these two approaches, with its 18.7% efficiency gain aligning closely with the green transition during high-quality development phases , thus reflecting more realistic developmental trajectories.

(2) Efficiency temporal evolution characteristics

All three methods show that the efficiency of China's provincial green economy exhibits a "U-shaped recovery-steady improvement" trend: during the stable growth period from 2014 to 2016, the average annual growth rate was 1.5%-2.1%, benefiting from the initial dividends of the digital economy; during the fluctuation adjustment period from 2017 to 2019, the volatility of DEA efficiency was significantly greater than that of StoNED, as DEA is sensitive to random shocks from extreme digital economy inputs and tightened environmental policies, while StoNED smooths out interference through dynamic trend terms and error decomposition; during the rapid improvement period from 2020 to 2024, the efficiency measured by StoNED reached 0.763 in 2024, an increase of 9.8% compared to 2020, reflecting the synergistic effect of deep integration between the "dual carbon" goals and the digital economy.

(3) Characteristics of regional differences

The empirical results demonstrate significant regional disparities that are converging. Eastern regions maintain sustained efficiency leadership through their advantages in digital economy and green technology innovation, while central and western regions achieve higher efficiency growth rates (21.3% in the west vs. 16.5% in the east) by undertaking industrial transfers and developing digital infrastructure. The East-West efficiency gap, measured by StoNED, has narrowed from 0.265 in 2014 to 0.208 in 2024, confirming the remarkable effectiveness of coordinated regional development during the high-quality development phase.

Table 4. Regional difference Characteristics of the three evaluation Methods

region	DEA mean	SFA mean	StoNED mean	Regional gap (east-west)
east	0.853	0.741	0.821	0.208
central	0.768	0.679	0.705	0.147
west	0.691	0.612	0.678	-

4.3. Empirical Test

(1) Correlation test

The Pearson correlation coefficients between the efficiency values of the three methods and the high-quality development index are as follows:

Table 5. Pearson correlation coefficients

correlation coefficient	DEA efficiency value	SFA efficiency value	StoNED efficiency value
High-quality Development Index	0.724(p<0.01)	0.689(p<0.01)	0.876(p<0.01)

The empirical results demonstrate that StoNED exhibits significantly stronger correlation with the high-quality development index, indicating its measurement better captures the core essence of high-quality development. In contrast, DEA and SFA show relatively weaker correlations due to insufficient consideration of new factors like the digital economy and stochastic environments.

(2) Stability test

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Firstly, after excluding five provinces with extreme digital economy performance such as Guangdong and Jiangsu, we conducted an outlier impact test. The results showed that the average change rate of DEA efficiency was 5.2%, SFA was 3.7%, and StoNED was only 1.9%, which confirmed the robustness of StoNED in dealing with extreme digital economy performance.

On the other hand, the sensitivity analysis of the function form was carried out by using C-D function and transcendental logarithm function to evaluate the efficiency of SFA, and the difference of the mean efficiency was 7.1%.

Finally, Monte Carlo simulations were conducted using 1000 generated random samples. The mean square error (MSE) of StoNED efficiency estimation (0.012) was significantly lower than that of DEA (0.038) and SFA (0.029), demonstrating its accuracy in stochastic environments.

(3) Rationality Test (Marginal Abatement Cost, MAC)

The marginal cost of CO_2 emission reduction was estimated through three methods and compared with the 2024 national carbon market average price (156 yuan/ton), as detailed in the table below.

Table 6. Marginal Cost of CO_2 Reduction and Comparative Analysis

method	MAC average (RMB/ton)	standard deviation	Carbon price deviation rate
DEA	132.8	41.5	-14.90%
SFA	221.6	36.8	42.10%
StoNED	163.5	27.3	4.80%

StoNED estimated MAC shows a mere 4.8% deviation from actual carbon market prices, significantly lower than DEA and SFA. This validates its validity in assessing green economic efficiency during high-quality development. Its Nash equilibrium-based shadow price estimation aligns with the carbon markets imperfect competition characteristics, providing a scientific basis for carbon pricing.

5. Conclusion

This study systematically compares three efficiency evaluation methods: non-parametric DEA, parametric SFA, and semi-parametric StoNED. Using panel data from 30 provincial-level administrative regions of China from 2014 to 2024 as samples, it empirically tests their suitability for evaluating green economic efficiency during the high-quality development stage, and draws the following core conclusions:

First, the three methods exhibit systematic differences in theoretical characteristics and empirical performance. The DEA method, while adaptable to multi-input and multi-output scenarios without requiring predefined production function forms, fails to account for random factors and is sensitive to outliers. The SFA method, though capable of distinguishing random errors from technical inefficiencies and providing statistical inference, relies on functional forms and distributional assumptions. The StoNED method, by constructing a non-parametric frontier through convex regression and incorporating composite error terms, resolves both DEAs oversight of random factors and SFAs dependence on functional forms. It demonstrates optimal performance across three dimensions-correlation, stability, and rationality-making it the ideal choice for evaluating green economic efficiency during the high-quality development phase.

Second, the empirical results show that China's green economic efficiency generally exhibited a "U-shaped recovery-steady improvement" trend from 2014 to 2024, with significant regional

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heterogeneity but a tendency toward convergence. The eastern region led in green economic efficiency, followed by the central region, while the western region lagged behind. After 2020, the deep integration of the "dual carbon" goals with the digital economy drove rapid efficiency improvements, verifying the critical role of policy and technology synergy.

Third, method selection significantly impacts efficiency evaluation outcomes. In practice, appropriate methods should be chosen based on research objectives and data characteristics. When focusing on statistical inference and random noise separation, the SFA method is more suitable; for multi-input multi-output systems, the DEA method demonstrates advantages; and when considering both method robustness and real-world complexity, the StoNED method proves ideal.

The contribution of this study lies in: constructing a green economic efficiency evaluation framework adapted to high-quality development, systematically verifying the superiority of the StoNED method; revealing the spatiotemporal evolution and regional convergence characteristics of China's green economic efficiency under the "dual carbon" goals; empirically testing for the first time the nonlinear threshold effect of the digital economy on green economic efficiency, providing scientific references for the formulation of regional differentiated policies. Future research can further expand the application scenarios of the method, deepen the analysis of driving mechanisms, and provide more precise theoretical support and practical guidance for high-quality development under the "dual carbon" goals.

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